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Status Seeking in the Small Open Economy

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Abstract

In our modified version of the small open economy Ramsey model, we assume that agents have preferences over consumption and status which, in turn, is determined by relative wealth. This extension potentially eliminates the standard model's counterfactual result that an impatient country over time mortgages all of its capital and labor income. We show that the steady-state values of net assets and consumption, the speed of convergence and, in particular, the direction of adjustment during the transition depend crucially upon the degree of status consciousness. The latter also influences the economy's response to macroeconomic shocks.

Keywords

Status seeking, relative wealth, open economy dynamics

JEL Classifications

E21, F41

Comments

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Contents

1 Introduction

An individual's utility is usually stated in terms of the *absolute* levels of economic variables, such as consumption of goods and services, leisure, wealth, etc. This standard specification is intuitively appealing and adequate to study many economic problems. There is evidence, some of which is provided by Easterlin (1974, 1995), Clark and Oswald (1996), Oswald (1997), and Frank (1997), to indicate, however, that an individual's economic well-being depends crucially on his relative position, or status, in society. The idea that individuals are motivated by status considerations is a very old one in economics and can be traced back to thinkers such as David Hume (1978) and Thorstein Veblen (1899). After World War II interest in this idea and its potential policy implications was maintained by authors such as Duesenberry (1949), Scitovsky (1976), Hirsch (1976), Boskin and Sheshinski (1978), Layard (1980), and Frank (1985a,b). In the last decade, there are an increasing number of researchers who study status preference in a dynamic macroeconomic or endogenous growth context. In general, there are two alternative ways in which status is modelled in macroeconomic settings. The approach adopted by Galí (1994), Persson (1995), Harbaugh (1996), Rauscher (1997b), Grossmann (1998), Ljungqvist and Uhlig (2000), and Fisher and Hof (2000) specifies that status derives from relative consumption. In contrast, Corneo and Jeanne (1997), Rauscher (1997a), Futagami and Shibata (1998), Fisher (2001), and Hof and Wirl (2001) consider that status arises from relative wealth.[1] While all these authors model the role of status preference in a closed economy context, we will introduce relative wealth into an otherwise standard small open economy Ramsey model. We believe this is an important extension of this line of research due to the increasing integration of the world economy and the greater role played by international assets in wealth accumulation.

Our work is, in addition, related to the recent Spirit of Capitalism literature, which is exemplified by authors such as Cole, Mailath, and Postlewaite (1992), Zou (1994, 1998) and Bakshi and Chen (1996), who seek to explain growth, savings, and asset pricing behavior. This research, based on the ideas of Max Weber (1958), views wealth accumulation as

[1] It is sometimes argued that because it is easier to "see" another person's level of consumption compared to his corresponding stock of assets, that the relative consumption approach is a more appropriate way to model status preference than the relative wealth approach. With, however, employee stock-options plans, among other forms of compensation, becoming more widespread, it may become easier to observe relative wealth than was previously the case.

the means to achieving social status, which itself enables agents to acquire nonmarket goods that are regarded as "prizes" by society at large. As stressed by Cole, Mailath, and Postlewaite (1992), utility functions that include variables such as relative wealth can then be interpreted as reduced-form versions of preferences over "deep" variables in which different social organizations can lead to different reduced-form preferences.

A further motivation for our approach is to offer an alternative solution to a long-standing issue in open economy macroeconomics: the fact that the representative agent model of the small open economy—under the assumption of perfect capital mobility—does not have a "sensible" steady-state equilibrium if the domestic rate of time preference differs from the world interest rate. For instance, if the exogenous rate of time preference of domestic residents exceeds the exogenous world interest rate, then agents eventually mortgage all of their capital and labor income. In contrast, if the economy is "more patient" than all others, it acquires over time the wealth of all other countries and, indeed, ceases to be a small open economy.[2] In order, then, for the small open economy to attain an interior equilibrium with a positive level of consumption, equality must be imposed between the domestic rate of time preference and the world interest rate. This condition, however, rules out the possibility of transitional dynamics, since it also fixes a particular stock of physical capital or assets. Turnovsky (1997) discusses several ways in which the standard small open economy Ramsey model can be extended to yield an *interior* long-run equilibrium and sensible transitional dynamics.[3] One approach that has been extensively used, [see, for instance, Brock (1988), Sen and Turnovsky (1989a, 1989b, 1990), and Frenkel, Razin, and Yuen (1996)], is to maintain the assumption that the domestic rate of time preference equals the world interest rate, but to additionally incorporate a convex installation cost function for domestic physical investment. This modification yields saddlepath dynamics for physical capital and its shadow value, but—due to the specified equality between the rate of time preference and the world interest rate—consumption equals its steady-state value for all time t, as in the standard model. If, however, labor supply is endogenously determined, then consumption does display saddlepath behavior

[2]See Barro and Sala-i-Martin (1995), chapter 3 for a discussion of these counterfactual cases.

[3]See Turnovsky (1997), chapters 2 and 3. He also points out that the potentially unpleasant features of the standard model do not occur in the usual overlapping generations framework or its Blanchard (1985) variant in which people, or dynasties, die off randomly. An extensive analysis of Blanchard's model is given in Barro and Sala-i-Martin (1995), chapter 3.

in this context. The adjustment cost approach is, nevertheless, not without its critics, who point out the difficulty in reconciling covariance properties of aggregate, postwar U.S. data with those of macroeconomic models that incorporate plausible values of the adjustment cost parameter.[4] Another, though more controversial, approach to generate sensible, long-run equilibria and saddlepath dynamics in the small open economy model is to specify that representative agents possess Uzawa (1968)–type, time-dependent, endogenous rates of time preference. This was the method used by Obstfeld (1982) in his work studying open economy dynamics. Authors such as Blanchard and Fischer (1989), Barro and Sala-i-Martin (1995) and Turnovsky (1997) find this formulation intuitively unappealing, however, because a necessary condition in the infinite horizon context to generate saddlepoint dynamics is to specify that the rate of time preference increases with the level of consumption. Obstfeld (1990), drawing on the work of Epstein (1987), among others, offers a defense of this approach and points out its usefulness in relaxing the more usual assumption of time-additive preferences, which is also restrictive.

Two additional, and related, approaches to addressing this issue are either to incorporate costs of holding foreign bonds or to specify an upward-sloping supply curve of debt. Both approaches attempt to model, in a certainty equivalence framework, the macroeconomic implications of imperfect substitutability between domestic and foreign assets. The first approach was taken by Turnovsky (1985), while the second has been adopted in Bhandari, Haque, and Turnovsky (1990). Using these specifications, interior solutions are obtained without imposing equality between the rate of time preference and the world interest rate. Because asset stocks will then adjust slowly according to international arbitrage, economies with these specifications exhibit saddlepath transitional dynamics. One issue that arises in these two cases is the fact that transitional dynamics can degenerate, depending on the macroeconomic shock in question.[5] In addition, it can be argued that it is better to model the implications of international capital market imperfections, which depend, at least in part, on the risk characteristics of domestic and foreign assets, in an explicitly stochastic setting.

Our model will offer an alternative way to generate interior long-run equilibria with

[4]This point is made by Kydland and Prescott (1982).

[5]Fisher (1995) and Fisher and Terrell (2000) show, however, that the upward-sloping debt specification is useful in calculating the implications of world interest rate disturbances.

positive consumption and sensible transitional dynamics for consumption and net assets. These properties arise due to the assumption that, in addition to own consumption, the instantaneous utility of agents is a function of status, which depends on relative wealth. Contrary to the existing literature, we will employ a specification of the status function that does not rule out—by definition—steady states with negative values of nonhuman wealth. In order to concentrate on the influence of status preference, we will abstract from population growth, technological progress, depreciation and installation costs of physical capital, and the heterogeneity of agents. Using an infinite-horizon, perfect-foresight framework, we will derive a symmetric equilibrium in which identical agents make the same choices. Our formulation of status preference will result in a modified version of the Euler equation. Its crucial feature is that the exogenous world interest rate is replaced by an endogenous, "effective" domestic rate of return that depends also on consumption and net assets. Weak assumptions with respect to preferences will yield saddlepoint dynamics. We will also incorporate a public sector in order to examine how the effects of (balanced-budget) fiscal policy are influenced by status preferences. In addition to changes in government expenditure, we will consider total factor productivity shocks.

The rest of the paper is organized as follows. Section 2, first describes our general model of status preference and derives the symmetric, intertemporal equilibrium. Along the stable arm, we show that consumption and net assets always move in the same direction. Moreover, as net assets accumulate, the effective rate of return declines, converging to the subjective discount rate, which is its long-run equilibrium value. We then parameterize the instantaneous utility function. This will allow us to compare the implications of different "degrees of status consciousness" for the steady-state values of consumption and net assets, as well as the speed of convergence. Moreover, we show that the degree of status consciousness determines whether net assets and consumption rise or fall during the transitional phase.

In section 3 we will investigate how status preference affects the adjustment of private consumption and net assets to government expenditure, and total factor productivity shocks. A notable result in this part of our paper is that in the long run, a rise in government expenditure "crowds out" private consumption by more than one-for-one. This is due to the long-run decline, attributable to status preference, in net assets, and hence, in net interest income. The crowding-out effect is more pronounced, the higher is the degree of status consciousness. We obtain an analogous result for a positive productivity shock in which the long-run rise in consumption exceeds that of after-tax real wage income due to higher steady-state net interest income. We close the paper with concluding remarks in section 4.

2 The Model and Intertemporal Equilibrium

2.1 General Specification of Preferences

We begin by assuming that the small open economy is populated by a large number of identical, infinitely-lived agents. Without loss of generality, we specify that the population size remains constant over time. In contrast to the standard model of the consumer, we assume that each agent possesses the following general instantaneous utility function over own consumption, c, and status, s, $U = U(c, s)$, where

$$U_c > 0, \quad U_s > 0, \quad U_{cc} < 0, \quad U_{ss} \leq 0, \quad U_{cc}U_{ss} - U_{cs}^2 \geq 0, \tag{1}$$

$$U_{sc}U_c - U_sU_{cc} > 0, \tag{2}$$

$$\lim_{c \to 0} U_c(c,\, s) = \infty, \qquad \lim_{c \to \infty} U_c(c,\, s) = 0. \tag{3}$$

According to (1), the representative agent derives positive, though diminishing, marginal utility from own consumption and positive and non-increasing marginal utility from status, with the utility function U jointly concave in c and s.[6] Condition (2) imposes normality on preferences, i.e., that the marginal rate of substitution of status for consumption, U_s/U_c, depends positively on c, while (3) describes the limiting behavior of the marginal utility of consumption. As indicated in the introduction, we assume that an individual's status depends on both own net assets (= nonhuman wealth), a, and average net assets of the private sector, A, i.e., $s = s(a, A)$, where the status function is defined for all $(a, A) \in (\bar{a}, \infty) \times (\bar{a}, \infty)$.[7] Since we do not want to rule out *a priori* the possibility that the economy reaches a steady state with a negative stock of private assets, as can be the

[6] We will use the following notational conventions. In general, we will suppress a variable's time dependence, i.e, $x \equiv x(t)$. The time derivative of x will be denoted by $\dot{x}$; a steady-state value by $\tilde{x}$. Unless otherwise indicated, the partial derivative of a function F with respect to x will be denoted by F_x.

[7] Subsequently, we will use "wealth" as a shorthand for "nonhuman wealth".

case in the standard open economy version of the Ramsey model, we will assume that $\bar{a} < 0$. This lower bound $\bar{a}$ can be interpreted as an indicator of domestic residents' aversion to (or tolerance for) indebtedness. In addition, we also assume for all $(a, A) \in (\bar{a}, \infty) \times (\bar{a}, \infty)$ that status increases in own wealth, decreases in average wealth, and that the marginal status in own assets is non-increasing, i.e.:

$$s_a > 0, \quad s_A < 0, \quad s_{aa} \leq 0. \tag{4}$$

In the bulk of macroeconomic literature on status preference, the status function takes the following "ratio" form $s(a, A) = \varphi(a/A)$, $\varphi' > 0$, $\varphi'' \leq 0$, where a/A represents relative wealth. Note, however, that this formulation yields counter-intuitive results if negative levels of wealth are permitted. For instance, since $s_a = A^{-1}\varphi'$, a negative level of average wealth ($A < 0$) would imply that an increase in own wealth *decreases* status. Similarly, $s_A = -(a/A^2)\varphi'$ implies that a rise in average wealth causes status to *improve* (i.e. $s_A > 0$) if the individual's wealth is negative ($a < 0$). To eliminate anomalies of this sort, we will use the following representation of the status function:

$$s(a, A) \equiv \varphi\left(\frac{a - \bar{a}}{A - \bar{a}}\right), \qquad \bar{a} < 0, \quad \varphi' > 0, \quad \varphi'' \leq 0. \tag{5}$$

According to (5), both own and average wealth are measured with respect to the "lower bound", or "minimum value", $\bar{a}$. It is easily verified that (5) satisfies all properties given in (4) for all $(a, A) \in (\bar{a}, \infty) \times (\bar{a}, \infty)$.

In order to focus on the influence of status preference, we make simple assumptions regarding the open economy's technological and financial market possibilities. Specifically, we assume that domestic physical capital is owned by domestic agents and is rented in the perfectly competitive global capital market. In addition to physical capital, they can accumulate wealth in the form of domestic government bonds (the domestic public sector will be introduced below) and international assets. Agents can also borrow in the international credit market. Own wealth a then consists of physical capital and net loans. Because physical capital and financial assets are perfect substitutes, they all bear the same rate of return, equal to the exogenous world interest rate r^*. Following Barro and Sala-i-Martin (1995) and Turnovsky (1997), we assume that r^* is constant through time. The representative agent inelastically supplies one unit of labor and receives a real wage income w that is determined in a perfectly competitive, domestic labor market. Assuming, in addition, that the domestic public sector levies per-capita lump-sum taxes τ on the private sector, we can express the flow budget constraint of the representative agent as

$$\dot{a} = r^* a + w - \tau - c, \tag{6}$$

where the agent's initial exogenous endowment of wealth is a_0. Employing an infinite horizon, perfect foresight framework, the agent's maximization problem is formulated as follows: maximize

$$\int_0^\infty U(c, s) e^{-\rho t}\, dt,$$

where ρ is the exogenous rate of pure time preference and s is given by equation (5), subject to the flow budget constraint (6), the initial condition $a(0) = a_0$, and the No-Ponzi-Game (NPG) condition $\lim_{t\to\infty} ae^{-r^* t} \geq 0$. A crucial feature of this optimization problem is that the representative agent takes the time path of average wealth A as given. In other words, each individual is small enough to neglect his own contribution to the average wealth level. The current value Hamiltonian for this problem is equal to

$$H(c, a, \lambda) = U\left(c, \varphi\left(\frac{a - \bar{a}}{A - \bar{a}}\right)\right) + \lambda(r^* a + w - \tau - c),$$

where λ is the current costate variable that denotes the current shadow value of wealth. The necessary conditions for an interior optimum, $H_c = 0$ and $\dot{\lambda} = \rho\lambda - H_a$, are then expressed as:

$$U_c\left(c, \varphi\left(\frac{a - \bar{a}}{A - \bar{a}}\right)\right) = \lambda, \tag{7}$$

$$\dot{\lambda} = (\rho - r^*)\lambda - U_s\left(c, \varphi\left(\frac{a - \bar{a}}{A - \bar{a}}\right)\right) \varphi'\left(\frac{a - \bar{a}}{A - \bar{a}}\right) \frac{1}{A - \bar{a}}. \tag{8}$$

The assumptions made above in (1) and (5) ensure that the Hamiltonian is jointly concave in the control variable c and the state variable a. This implies that if the limiting transversality condition $\lim_{t\to\infty} \lambda a e^{-\rho t} = 0$ holds, then the necessary conditions (7)–(8) are sufficient for optimality.

We now describe the domestic public sector and assume, first, that it has the following flow budget constraint $\dot{b} = r^* b + g - \tau$, where b is the stock of per-capita government debt, g represents per-capita government spending, and, as indicated above, τ is the level of per-capita lump-sum taxes. The government can borrow from the domestic private sector or from abroad at the prevailing world interest rate r^*. We further assume that the accumulation of government debt is subject to the following NPG condition $\lim_{t\to\infty} be^{-r^*t} = 0$. For simplicity, we assume that the fiscal policy variables g and τ are constant through time. In this special case, the NPG condition imposes the balanced-budget rule $\tau = g + r^*b_0$, $\forall t \geq 0$, so that the stock of government debt remains at its initial value, b_0.

The next step is to derive the intertemporal macroeconomic equilibrium. Following the standard procedure for models of status preference with homogeneous agents, we restrict our analysis to symmetric equilibria in which identical agents make identical choices. Consequently, $a = A$ holds $\forall t \geq 0$. From $s(a,a) = \varphi(1)$, it follows that in any symmetric equilibrium, the flow of utility, $U(c, \varphi(1))$, is independent of the common level of wealth.[8] As indicated, a, the wealth of the domestic private sector, consists of physical capital and net claims on the domestic government and on the rest of the world. By definition, the overall per-capita *net foreign asset position* of the open economy is then given by $(a - b_0 - k)$, where k denotes the domestic stock of physical capital per person. (For the remainder of the paper, lower-case variables will refer to their economy-wide, per-capita levels.) In this standard framework, the stock of capital held, k, and the market-clearing real wage, w, are determined by the usual profit-maximizing conditions. If the (per-capita) production function takes the form $B \cdot f(k)$, where B denotes total factor productivity, and $f(k)$ satisfies the standard neoclassical properties, which are given by $f'(k) > 0$, $f''(k) < 0$, $f(0) = 0$, $f(k) \to \infty$ as $k \to \infty$ and the Inada conditions, then $r^* = Bf'(k)$ and $w = B[f(k) - kf'(k)]$. Since both B and r^* are time invariant by assumption, the equilibrium values of the capital stock and real wage will not exhibit any transitional behavior. In other words, the capital stock and the real wage always equal their steady-state values $\tilde{k}$ and $\tilde{w}$. It is straightforward to show that both $\tilde{k}$ and $\tilde{w}$ depend negatively on the world interest rate r^* and positively on total factor productivity B:

$$\tilde{k} = \tilde{k}(r^*, B), \qquad \tilde{k}_{r^*} = [Bf''(\tilde{k})]^{-1} < 0, \qquad \tilde{k}_B = -f'(\tilde{k})[Bf''(\tilde{k})]^{-1} > 0,$$

$$\tilde{w} = \tilde{w}(r^*, B), \qquad \tilde{w}_{r^*} = -\tilde{k} < 0, \qquad \tilde{w}_B = f(\tilde{k}) > 0. \tag{9}$$

Substitution of $a = A$, $w = \tilde{w}$, and $\tau = r^*b_0 + g$ into the two optimality conditions (7)–(8) and asset accumulation equation (6) results in the following balanced-budget, symmetric

[8] This property would not be preserved if (5) were replaced by $s(a, A) \equiv \varphi((a - \bar{a})/(A - \bar{A}))$, $\bar{a} \neq \bar{A}$.

open economy equilibrium in which the paths of c, a, and λ obey the following relationships

$$U_c(c,\varphi(1)) = \lambda, \tag{10}$$

$$\dot{\lambda} = (\rho - r^*)\,\lambda - U_s(c,\varphi(1))\frac{\varphi'(1)}{a-\bar{a}}, \tag{11}$$

$$\dot{a} = r^* a + \tilde{w} - (g + r^* b_0) - c, \tag{12}$$

as well as the initial condition $a(0) = a_0$ and the transversality condition $\lim_{t\to\infty} \lambda a e^{-\rho t} = 0$. Note that equation (12) corresponds to the current account balance.[9]

It is useful to represent the dynamic system in terms of the control variable consumption, rather than the shadow value of wealth λ, since this will give us an Euler equation that we can directly compare to the Euler equation implied by the standard model in which the quest for status does not take place. Taking the time derivative of (10), yields $\dot{\lambda} = U_{cc}(c,\varphi(1))\dot{c}$. Substituting this expression into (11) and using (10) results in the following modified Euler equation

$$\dot{c} = c\sigma^e(c)[r^e(r^*,c,a) - \rho], \tag{13}$$

where the *effective* elasticity of intertemporal substitution σ^e and the *effective* domestic, or internal, rate of return on assets r^e, respectively, are given by

$$\sigma^e(c) \equiv -\frac{U_c(c,\varphi(1))}{cU_{cc}(c,\varphi(1))}, \qquad r^e(r^*,c,a) \equiv r^* + \frac{U_s(c,\varphi(1))}{U_c(c,\varphi(1))}\,\frac{\varphi'(1)}{a-\bar{a}}\,. \tag{14}$$

Using the fact that

$$\frac{U_s(c,\varphi(1))}{U_c(c,\varphi(1))}\,\frac{\varphi'(1)}{a-\bar{a}} = \frac{U_s(c,\varphi(1))}{U_c(c,\varphi(1))}\,s_a\,(a,a) = \frac{U_s(c,s(a,A))}{U_c(c,s(a,A))}\,s_a(a,A)\Big|_{a=A},$$

it follows that the effective rate of return r^e equals the sum of the world interest rate r^* and the marginal rate of substitution of own assets a for consumption c as perceived by the representative agent in a symmetric state in which $a = A$ holds (hereafter, symmetric MRS). In general, the MRS of a for c represents the additional return to saving due to status preference in which the incremental flow of utility from an extra unit of savings, equal to $U_s(c,s(a,A))s_a(a,A)$, is converted, through division by $U_c(c,s(a,A))$, into equivalent units of the consumption good. Differentiating the expression for $r^e(r^*,c,a)$ with respect

[9]The current account balance equals the rate of change of the open economy's net foreign asset position, $d(a-b-k)/dt$. Since, however, $b = b_0$ and $k = \tilde{k}$, $\forall t \geq 0$, the current account balance reduces to $\dot{a}$.

to c and a, it is straightforward to show that the symmetric MRS, and hence the effective rate of return, is a positive function of consumption and a negative function of assets. The partial derivatives of the effective rate of return with respect to consumption and assets are given, respectively, by:

$$r_c^e(c,a) = \frac{U_{sc}(c,\varphi(1))U_c(c,\varphi(1)) - U_s(c,\varphi(1))U_{cc}(c,\varphi(1))}{[U_c(c,\varphi(1))]^2} \frac{\varphi'(1)}{a-\bar{a}} > 0, \tag{15}$$

$$r_a^e(c,a) = -\frac{U_s(c,\varphi(1))}{U_c(c,\varphi(1))} \frac{\varphi'(1)}{(a-\bar{a})^2} < 0. \tag{16}$$

The positive sign of r_c^e is due to the normality assumption stated in (2), which states that the MRS of status for consumption, $U_s(c,s)/U_s(c,s)$, is an increasing function of consumption c for any given value of status s. In contrast, the negative sign of r_a^e depends on the specification of the status function (5). In particular, since

$$\frac{ds_a(a,a)}{da} = s_{aa}(a,a) + s_{aA}(a,a) = -\frac{\varphi'(1)}{(a-\bar{a})^2} < 0,$$

it follows that if the economy moves from one symmetric state to another in which each individual has a greater level of wealth, then the marginal status of own wealth, given by $s_a(a,a)$, and, hence, the symmetric MRS of a for c declines for any value of c. Finally, we can show that the transversality condition in this context can be re-expressed as:[10]

$$\lim_{t\to\infty} \left\{ a\,(t) \exp\left[-\int_0^t r^e(r^*, c(v), a(v))\, dv \right] \right\} = 0.$$

Since the effective rate of return $r^e(r^*,c,a)$ is not fixed parametrically, it is possible in our model for the small open economy to have saddlepoint stable transitional dynamics. By way of contrast, assume, instead, that agents have standard concave preferences solely over own consumption, i.e., $U = U(c)$, $U' > 0$, $U'' < 0$. The Euler equation (13) then collapses to $\dot{c} = c\sigma(c)(r^* - \rho)$, where $\sigma(c) \equiv -U'(c)/[cU''(c)]$ is the standard elasticity of substitution and where $\lim_{t\to\infty} ae^{-r^*t} = 0$ becomes the appropriate transversality condition in this case. Since r^* and ρ are both fixed constants, the standard Euler equation implies that for the small open economy to reach an interior long-run equilibrium with positive consumption, the equality $r^* = \rho$ must be imposed. But this means, however, that the small open economy exhibits no transitional dynamics, since $\dot{c} = \dot{a} = 0$, $\forall t \geq 0$, in this case. If, on

[10] Because we are only concerned with steady states in which a takes a finite value, the modified transversality condition implies that the NPG holds with equality.

the other hand, $r^* \neq \rho$, the small open economy exhibits the "counter-intuitive" dynamic behavior we described in the introduction: either mortgaging all its net labor income if $r^* < \rho$ or eventually ceasing to be a small economy if $r^* > \rho$.[11] In our model of status preference in which consumption dynamics depends on the effective rate of return, agents are, by assumption, impatient in the sense that $r^* < \rho$. Nevertheless, we will show that the small open economy possesses steady states with positive levels of consumption without eliminating transitional dynamic behavior.[12]

It is instructive at this point in the paper to compare the effective rate of return in equation (14) with the upward-sloping debt function discussed in the introduction. In general, this relationship assumes the following functional form $r(z) = r^* + \psi(z)$, where $z \equiv -(a - b_0 - k)$ represents the stock of net international debt, and $\psi(\cdot) > 0$, $\psi' > 0$, $\psi'' > 0$. The domestic interest rate $r(z)$ equals the sum of the world interest rate r^* and the country-specific "risk premium" $\psi(z)$ that increases with z.[13] Observe that our formulation of the effective rate of return in equation (14) and the upward-sloping debt function have two common properties. First, according to both relationships an increase in indebtedness raises the effective domestic rate of return on saving. Second, in a steady state without endogenous growth $r^* + \psi(\tilde{z}) = \rho$ with $\psi(\tilde{z}) > 0$ will obtain. Hence, as in our model, a necessary condition for the existence of such a steady state is that domestic agents are impatient in the sense that $r^* < \rho$. Nevertheless, the upward-sloping debt function does not explicitly arise—unlike the effective rate of return in (14)—from the optimizing behavior of agents and, in particular, does not depend on preferences and consumption. This distinction is important (see footnote 26) in determining the response of the economy, for example, to fiscal shocks.

[11]If $r^* < \rho$, the optimal paths of assets and consumption exhibit the following limiting behavior:

$$\lim_{t\to\infty} a(t) = -\frac{\tilde{w} - r^* b_0 - g}{r^*} = -\frac{\tilde{w} - \tau}{r^*} < 0, \qquad \lim_{t\to\infty} c(t) = 0.$$

[12]It must be stressed that our results depend critically on the fact that status is a function of relative wealth. If status is, instead, a function of relative consumption, i.e., $s(c, C) = \varphi(c/C)$, where C represents average consumption, the Euler equation becomes $\dot{c} = c\sigma^{e,c}(c)(r^* - \rho)$, where the effective elasticity of intertemporal substitution $\sigma^{e,c}(c)$ takes a form distinct from the expression in equation (14) [for mathematical details, see Fisher and Hof (2000)]. Since the effective rate of return is simply the world interest rate r^* in the relative consumption case, the economy would be in either of the two counter-intuitive cases described above, unless $r^* = \rho$.

[13]Alternative formulations of this relationship scale net debt by output or the capital stock.

In terms of the Uzawa model, the economy's consumption dynamics depend [see, for instance, Turnovsky (1997)] on what can be called the economy's *effective rate of time preference*, i.e., $\rho^e = \rho\left(U(c)\right)$, where $\rho' > 0$, $\rho'' > 0$, $\rho - U(c)\rho' > 0$. As we discussed in the introduction, the effective rate of time preference in the Uzawa framework is a positive function of flow utility, which, in turn, depends positively on own consumption c. The positive relationship between ρ^e and c is a necessary condition for the economy to possess a saddlepoint steady state. Because the relationship $\rho\left(U(\tilde{c})\right) = r^*$ will hold in the steady state, one potential limitation of the Uzawa model in the small open economy context is that long-run value of consumption $\tilde{c}$ will depend—aside from the properties of the function $\rho(\cdot)$—solely, and positively, on the world interest rate r^*. This implies that neither changes in domestic fiscal policy nor productivity shocks will affect $\tilde{c}$ in the Uzawa model.

Returning to our analysis, we next consider steady states $(\tilde{a}, \tilde{c})$ of the small open economy in which the long-run level of consumption is strictly positive, $c = \tilde{c} > 0$. In graphical terms—with a plotted on the horizontal axis and c plotted on the vertical axis in Figure 1—these are determined by the points of intersections between the $\dot{a} = 0$ line, given by

$$c = r^* a + \tilde{w} - (r^* b_0 + g), \tag{17}$$

and the $\dot{c} = 0$ locus implicitly defined by $r^e(r^*, c, a) = \rho$. Observe that the latter can be rewritten as:

$$\frac{U_s(c, \varphi(1))}{U_c(c, \varphi(1))} \frac{\varphi'(1)}{a - \bar{a}} = \rho - r^*, \tag{18}$$

where the left-hand side of (18) is the symmetric MRS of own assets for consumption and, as indicated, $(\rho - r^*) > 0$ holds by assumption. We know from the above analysis that r^e and the symmetric MRS depend positively on c. If, in addition, the symmetric MRS satisfies the weakened Inada conditions

$$\lim_{c \to 0} \left[\frac{U_s(c, \varphi(1))}{U_c(c, \varphi(1))} \frac{\varphi'(1)}{a - \bar{a}} \right] < \rho - r^* < \lim_{c \to \infty} \left[\frac{U_s(c, \varphi(1))}{U_c(c, \varphi(1))} \frac{\varphi'(1)}{a - \bar{a}} \right], \quad a \in (\bar{a}, \infty),$$

which are equivalent to $\lim_{c \to 0} r^e(r^*, c, a) < \rho < \lim_{c \to \infty} r^e(r^*, c, a)$, then for any given value of net assets $a \in (\bar{a}, \infty)$, there exists a unique positive value of consumption c that solves $r^e(r^*, c, a) = \rho$. In graphical terms, this means that the $\dot{c} = 0$ locus does not intersect the horizontal axis within the interval $(\bar{a}, \infty)$. Since $r_c^e > 0$ and $r_a^e < 0$, $(dc/da)|_{\dot{c}=0} = -r_a^e/r_c^e > 0$, i.e., the $\dot{c} = 0$ locus is positively sloped in the (a, c)–space.[14] Moreover, it is obvious from equation (17) that the $\dot{a} = 0$ line has a constant slope equal to the world interest rate, i.e., $(dc/da)|_{\dot{a}=0} = r^* > 0$, with a horizontal intercept given by $-(\tilde{w} - r^* b_0 - g)/r^* = -(\tilde{w} - \tau)/r^*$. The following stability analysis will demonstrate that a steady state $(\tilde{a}, \tilde{c})$— in which $\tilde{c} > 0$— is a saddlepoint equilibrium if and only if $\dot{c} = 0$ locus cuts the $\dot{a} = 0$ from below at $(\tilde{a}, \tilde{c})$.

Linearizing the system (12)–(13) about the steady state $(\tilde{a}, \tilde{c})$, the dynamics can be approximated by the following matrix differential equation:

$$\begin{pmatrix} \dot{a} \\ \dot{c} \end{pmatrix} = \begin{pmatrix} r^* & -1 \\ \tilde{c}\sigma^e(\tilde{c}) r_a^e(\tilde{c}, \tilde{a}) & \tilde{c}\sigma^e(\tilde{c}) r_c^e(\tilde{c}, \tilde{a}) \end{pmatrix} \begin{pmatrix} a - \tilde{a} \\ c - \tilde{c} \end{pmatrix}. \tag{19}$$

The stability properties of (19) can be investigated by examining its characteristic polynomial, which is given by

$$0 = (r^* - \mu)[\tilde{c}\sigma^e(\tilde{c}) r_c^e(\tilde{c}, \tilde{a}) - \mu] + \tilde{c}\sigma^e(\tilde{c}) r_a^e(\tilde{c}, \tilde{a}) \tag{20}$$

$$= \mu^2 - [\mathrm{tr}\,(\mathbf{J})]\mu + \det(\mathbf{J}), \tag{21}$$

where the trace and determinant of the Jacobian matrix $\mathbf{J}$ in (19) are given by

$$\mathrm{tr}\,(\mathbf{J}) = r^* + \tilde{c}\sigma^e(\tilde{c}) r_c^e(\tilde{c}, \tilde{a}) > 0, \qquad \det(\mathbf{J}) = \tilde{c}\sigma^e(\tilde{c})[r^* r_c^e(\tilde{c}, \tilde{a}) + r_a^e(\tilde{c}, \tilde{a})]. \tag{22}$$

As is well-known, the eigenvalues μ_1, μ_2 satisfy both $\mathrm{tr}\,(\mathbf{J}) = \mu_1 + \mu_2$ and $\det(\mathbf{J}) = \mu_1\mu_2$.[15] For a steady state $(\tilde{a}, \tilde{c})$ to be a saddlepoint, $\det(\mathbf{J})$ must be negative and, consequently, so must be the term in brackets in (22). As illustrated in Figure 1, this is the case as long as the $\dot{c} = 0$ locus cuts the $\dot{a} = 0$ locus from below at the steady-state equilibrium $(\tilde{a}, \tilde{c})$. That is:

$$\det(\mathbf{J}) < 0 \Leftrightarrow \left.\frac{dc}{da}\right|_{a=\tilde{a},\ \dot{c}=0} = \frac{-r_a^e(\tilde{c}, \tilde{a})}{r_c^e(\tilde{c}, \tilde{a})} > r^* = \left.\frac{dc}{da}\right|_{\dot{a}=0}.$$

[14] Figure 1 depicts the special case in which the $\dot{c} = 0$ locus describes a linear relationship. This need not be the case for the general model of preferences.

[15] From $r_c^e > 0$ and $r_a^e < 0$ it follows that

$$[\mathrm{tr}\,(\mathbf{J})]^2 - 4\det(\mathbf{J}) = [r^* - \tilde{c}\sigma^e(\tilde{c}) r_c^e(\tilde{c}, \tilde{a})]^2 - 4\tilde{c}\sigma^e(\tilde{c}) r_a^e(\tilde{c}, \tilde{a})$$

is strictly positive. Hence, both eigenvalues are real numbers, irrespective of the sign of $\det(\mathbf{J})$.

Under this condition, the steady-state equilibrium $(\tilde{a}, \tilde{c})$ is a saddlepoint, with $\mu_1 < 0$, $\mu_2 > 0$, $|\mu_1| < \mu_2$.[16] Using standard methods, we then obtain the following linearized solution for assets and the stable saddlepath describing the dynamics of (a, c)

$$a = \tilde{a} - (\tilde{a} - a_0)e^{\mu_1 t},$$

$$c - \tilde{c} = (r^* - \mu_1)(a - \tilde{a}) = -\frac{\tilde{c}\sigma^e(\tilde{c})r^e_a(\tilde{c}, \tilde{a})}{\tilde{c}\sigma^e(\tilde{c})r^e_c(\tilde{c}, \tilde{a}) - \mu_1}(a - \tilde{a}), \tag{23}$$

where a adjusts from an exogenous initial stock, a_0.[17] The stable saddlepath, denoted by XX in Figure 1, is positively sloped, which implies that consumption and wealth move in the same direction, i.e., $\operatorname{sgn}(\dot{c}) = \operatorname{sgn}(\dot{a})$. These relationships are illustrated in Figure 1, which is the phase diagram of the dynamic system using our general specification of preferences.[18] In Figure 1 the long-run saddlepoint equilibrium is given by point D and corresponds to a positive steady-state level of assets. Observe, in addition, that we have indicated an initial equilibrium $(a_0, c(0))$ along the saddlepath XX that lies to the north-east of D, which implies that the economy will run current account deficits in transition to point D.

We can also use our solution for the stable saddlepath XX to describe the dynamic adjustment of the effective rate of return $r^e(r^*, c, a)$. At first glance, whether r^e rises or falls towards its steady-state value ρ appears ambiguous, since while $\operatorname{sgn}(\dot{c}) = \operatorname{sgn}(\dot{a})$ along XX, the partial derivatives r^e_c and r^e_a are of opposite sign, i.e., $r^e_c > 0$, $r^e_a < 0$. Nevertheless, linearizing $r^e(r^*, c, a)$ about the steady-state equilibrium, substituting for $(c - \tilde{c})$ from (23) in the resulting expression and using (20), we obtain the following relationship between r^e and a:

$$r^e(r^*, c, a) - \rho = \frac{(r^* - \mu_1)\mu_1}{\tilde{c}\sigma^e(\tilde{c})}(a - \tilde{a}) = -\frac{\mu_1 r^e_a(\tilde{c}, \tilde{a})}{\tilde{c}\sigma^e(\tilde{c})r^e_c(\tilde{c}, \tilde{a}) - \mu_1}(a - \tilde{a}). \tag{24}$$

This equation implies that if $a < \tilde{a}$ (resp. $a > \tilde{a}$), then $r^e > \rho$ (resp. $r^e < \rho$) along the stable arm. During the transition to the steady state r^e and a move in opposite directions,

[16] The opposite case in which the $\dot{a} = 0$ locus cuts the $\dot{c} = 0$ locus from below implies, in contrast, that the steady state $(\tilde{a}, \tilde{c})$ is an unstable node with $0 < \mu_1 < \mu_2$.

[17] The two equivalent representations of the saddlepath in (23) are derived from (20).

[18] In Figure 1 the arrows can be interpreted as follows: above the $\dot{c} = 0$ locus, $r^e > \rho$ obtains. According to (13), it is optimal for the agent to choose a rising consumption path, i.e., $\dot{c} > 0$. Above the $\dot{a} = 0$ locus, the agent dissaves, i.e., $\dot{a} < 0$, since consumption exceeds the sum of after-tax wage income and net interest income. Conversely, $\dot{c} < 0$ below the $\dot{c} = 0$ locus, and $\dot{a} > 0$ below the $\dot{a} = 0$ locus.

i.e., $\operatorname{sgn}(\dot{r}^e) = -\operatorname{sgn}(\dot{a})$. This suggests that the influence of net assets on the effective of return dominates that of consumption after $t > 0$.

2.2 Parameterized Model

To describe in further detail the role of status in influencing the dynamic equilibrium, it is convenient to parameterize the preferences of the representative agent. We use a generalized version of the preferences employed by Futagami and Shibata (1998) and assume that the instantaneous utility function takes the following functional form

$$U(c,s) = \frac{1}{1-\theta}\left[\left(c^{\eta}s^{\beta}\right)^{1-\theta} - 1\right], \tag{25}$$

where $\eta > 0$, $\beta > 0$, $\theta > 0$, $1-\eta(1-\theta) > 0$, $1-\beta(1-\theta) \geq 0$, $1-(\beta+\eta)(1-\theta) \geq 0$. However, the form of $U(c,s)$ introduced in (25) requires the additional assumption of $\varphi > 0$ in order to ensure that $s > 0$.[19] Our assumptions guarantee that (25) satisfies the restrictions given in (1)–(3). Under this specification, the parameterized expressions for the effective elasticity of intertemporal substitution and the effective rate of return are equal, respectively, to $\sigma^e = [1-\eta(1-\theta)]^{-1} > 0$ and

$$r^e = r^* + (\xi/\eta)\frac{c}{a-\bar{a}}, \qquad \text{where } \xi \equiv \frac{\beta\varphi'(1)}{\varphi(1)}. \tag{26}$$

From (26), note that the parameter ξ depends positively on β as well as on the derivative $\varphi'(1)$. The higher is ξ, the more important is asset accumulation for status-seeking individuals.[20] Observe, in addition, that the intertemporal elasticity of substitution σ^e is independent of ξ as well as independent of the level of consumption c. From (26), it follows that the $\dot{c} = 0$ locus, implicitly defined by $r^e = \rho$, assumes the following linear form:

$$c = (\eta/\xi)(\rho - r^*)(a - \bar{a}). \tag{27}$$

Observe that the $\dot{c} = 0$ locus will cut the a-axis at $\bar{a}$. This reflects that fact as $a \to \bar{a}$, $c \to 0$. Clearly, the $\dot{c} = 0$ locus is "flatter", the greater is the parameter ξ that scales the importance of status in our model. Recalling from (17) that the $\dot{a} = 0$ line corresponds to

[19] In the closed-economy model studied by Futagami and Shibata (1998) $\bar{a} = 0$.

[20] The intuition is clear: the greater is β, the higher is the MRS of status for private consumption $[U_s(c,s)/U_s(c,s) = (\beta/\eta)(c/s)]$ and the greater is the derivative $\varphi'(1)$, the higher is the marginal status of own assets as perceived by the representative agent in symmetric states $[s_a(a,a) = \varphi'(1)(a-\bar{a})^{-1}]$.

the equation $c = r^*a + \tilde{w} - g - r^*b_0$, we can show that the expressions for the steady-state values of consumption $\tilde{c}$ and the excess of assets over their lower bound $(\tilde{a} - \bar{a})$ are equal to:

$$\tilde{a} - \bar{a} = \frac{(\xi/\eta)r^*}{\rho - [1 + (\xi/\eta)]r^*} \left[\bar{a} + \frac{\tilde{w} - r^*b_0 - g}{r^*} \right], \tag{28}$$

$$\tilde{c} = \frac{(\rho - r^*)\, r^*}{\rho - [1 + (\xi/\eta)]r^*} \left[\bar{a} + \frac{\tilde{w} - r^*b_0 - g}{r^*} \right]. \tag{29}$$

For the remainder of the paper, we will restrict our analysis to the case in which preferences obey the following two conditions:

$$\bar{a} > -\frac{\tilde{w} - r^*b_0 - g}{r^*} = -\frac{\tilde{w} - \tau}{r^*}, \qquad \rho - [1 + (\xi/\eta)]r^* > 0. \tag{30}$$

If the restrictions in (30) are met, then the steady state $(\tilde{a}, \tilde{c})$ described by the solutions (28)–(29) satisfies the following properties: i) it is economically sensible in the sense that consumption $\tilde{c}$ is positive and the stock of assets $\tilde{a}$ exceeds its lower bound $\bar{a}$, ii) it is a saddlepoint. The first condition in (30) requires that the lower bound on assets exceeds the negative of the steady-state value of discounted, after-tax wage income. Graphically, it ensures that the point of intersection of the $\dot{c} = 0$ locus with the a-axis lies to the right of the corresponding point of intersection of the $\dot{a} = 0$ locus. The second condition in (30) places, in effect, a lower bound on the rate of time preference ρ for given values of ξ, η, and r^*. Equally, this condition—if rewritten as $\xi < (\eta/r^*)(\rho - r^*)$—can be interpreted as imposing an upper bound on ξ for given values of ρ, η, and r^*. In Fig. 1, it guarantees that the $\dot{c} = 0$ locus is steeper than the $\dot{a} = 0$ locus.[21]

We close this subsection with an analysis of the role played by the status parameter ξ in determining the direction of the economy's adjustment from a given initial stock of assets a_0. The role of ξ in influencing the economy's speed of adjustment will also be considered. Figure 2 illustrates two possible paths consumption and net assets can take starting from a given positive value of a_0, under the assumption that all parameters are the same *except* for the status parameter ξ. This implies that the $\dot{a} = 0$ locus will be identical in both cases, while the $\dot{c} = 0$ locus will differ only according to its slope, which, as indicated, depends negatively on the value of ξ. In Figure 2 the saddlepath EF corresponds to a

[21] If the signs of the restrictions in (30) were both reversed, the steady state would be meaningful in the sense that $\tilde{c} > 0$ and $(\tilde{a} - \bar{a}) > 0$, although the long-run equilibrium would be an unstable node. If, instead, only one of the conditions is violated, then a sensible steady-state equilibrium will not exist.

"low" value of ξ, denoted by ξ_{low}, and takes the economy to a steady-state equilibrium in which the stock of net assets of the becomes negative, i.e., $\tilde{a} < 0$. In contrast, the path $E'F'$ corresponds to a "high" value of ξ, denoted by ξ_{high}.[22] Along the saddlepath $E'F'$ the economy will run current account surpluses that will improve the net asset position of the private sector to $\tilde{a}'$ in the long run, which will also lead to a higher level $\tilde{c}'$ of steady-state consumption.[23] Graphically, the ξ_{low} case in Figure 2 is illustrated by the relatively steep $\dot{c}\,(\xi_{low}) = 0$ locus, while the corresponding ξ_{high} case is depicted by its flatter counterpart and is denoted by $\dot{c}\,(\xi_{high}) = 0$.

To consider the way in which status preference affects the speed of adjustment along the saddlepath, we will make use of the characteristic polynomial. Substituting the appropriate expressions for σ^e, r_a^e, and r_c^e into equations (22), we derive the following parameterized expressions for the trace and the determinant of the Jacobian matrix $\mathbf{J}$:

$$\operatorname{tr}(\mathbf{J}) = r^* + \frac{\rho - r^*}{1 - \eta(1-\theta)}, \qquad \det(\mathbf{J}) = -\frac{(\rho - r^*)\{\rho - [1 + (\xi/\eta)]r^*\}}{[1 - \eta(1-\theta)](\xi/\eta)} < 0.$$

From (23), it is clear that the slope of the stable saddlepath, $r^* - \mu_1$, becomes "steeper", the larger in absolute value is the negative eigenvalue μ_1. The question remains how $|\mu_1|$ is affected by the parameter ξ. It is straightforward to calculate that $\partial|\mu_1|/\partial\xi < 0$. In other words, an increase in the parameter ξ, corresponding to an increase in the importance of status, reduces the stable speed of adjustment $|\mu_1|$. Consequently, not only is the slope of the $\dot{c} = 0$ locus reduced by an increase in ξ, but so also is the slope of the saddlepath. We depict this in Figure 2 where the saddlepath $E'F'$ corresponding to the ξ_{high} case is flatter than the saddlepath EF corresponding to the ξ_{low} case. As we shall see in the next section, a higher of ξ can also have the effect of "magnifying" the steady-state response of the economy to government expenditure and total factor productivity shocks.

3 The Role of Status in Macroeconomic Adjustment

In this section we will describe the role of status in determining the response of the small open economy to the changes (shocks) in the levels of government expenditure and total

[22] The "prime" notation here denotes the alternative equilibrium with a high value of ξ, ξ_{high}. In section 3, Figures 3 and 4, the variable $\tilde{x}'$ will denote the new, long-run value of a variable x in response to a permanent shift in government expenditure or total factor productivity.

[23] There will in general exist a critical value of ξ for which $\tilde{a} = 0$.

factor productivity. We will only consider shocks that are permanent and unanticipated. This implies that the change in the relevant exogenous variables will be implemented at $t = 0$, with the exogenous variable remaining forever at its new, higher level as the economy moves along its perfect foresight saddlepath. In our subsequent discussion, we will present the steady-state comparative expressions of both the general and the parameterized model, since we can extract insights from both representations. We will focus on the response of net assets $\tilde{a}$ and consumption $\tilde{c}$. The comparative statics relationships of the general model will be calculated using the following steady-state system of equations

$$\tilde{c} = r^* \tilde{a} + \tilde{w} - (r^* b_0 + g), \qquad r^e(r^*, \tilde{c}, \tilde{a}) = \rho, \tag{31}$$

where $\tilde{w} = \tilde{w}(r^*, B)$. Note that (31) corresponds to (17)–(18)—the equations describing the $\dot{a} = 0$ and $\dot{c} = 0$ loci—expressed in terms of the steady-state values of $\tilde{a}$ and $\tilde{c}$. To determine the comparative statics expressions of the parameterized model, we will use the solutions (28)–(29) for $(\tilde{a} - \bar{a})$ and $\tilde{c}$.

3.1 Fiscal Shocks

We discuss in this subsection the effect of a permanent increase in domestic government expenditure on the steady-state values of net assets and consumption. In the general preference framework, differentiation of (31) with respect to g yields

$$\frac{\partial \tilde{a}}{\partial g} = -\frac{r_c^e(\tilde{c}, \tilde{a})}{\Psi} < 0, \qquad \frac{\partial \tilde{c}}{\partial g} = \frac{r_a^e(\tilde{c}, \tilde{a})}{\Psi} = -\left[1 + \frac{r^* r_c^e(\tilde{c}, \tilde{a})}{\Psi}\right] < -1, \tag{32}$$

where $\Psi = -\left[\tilde{c}\sigma^e(\tilde{c})\right]^{-1} \det(\mathbf{J}) > 0$. The inequalities in (32), which state that an increase in government expenditure decreases the steady-state stock of net assets and the level of consumption, follow from $r_c^e > 0$ and $r_a^e < 0$, along with the fact that we only consider saddlepoint-stable long-run equilibria. In the parameterized model, the corresponding expressions are equal to:[24]

$$\frac{\partial \tilde{a}}{\partial g} = -\frac{\xi/\eta}{\rho - \left[1 + (\xi/\eta)\right] r^*} < 0, \qquad \frac{\partial \tilde{c}}{\partial g} = -\frac{\rho - r^*}{\rho - \left[1 + (\xi/\eta)\right] r^*} < -1. \tag{33}$$

It is clear that the absolute values of the government expenditure multipliers for net assets and consumption, $|\partial \tilde{a}/\partial g|$ and $|\partial \tilde{c}/\partial g|$, depend positively on the status parameter

[24]The inequalities in (33) follow from our assumption $(\rho - r^*) > 0$ and the second condition in (30).

ξ. Hence, the long-run decline in net assets and consumption is greater, the larger is ξ. As we have discussed, a larger value of ξ corresponds graphically to a "flatter" $\dot{c} = 0$ locus. A flatter $\dot{c} = 0$ locus implies, in turn, that greater changes in the values of $\tilde{c}$ and $\tilde{a}$ take place subsequent to a given shift in the $\dot{a} = 0$ locus. Furthermore, observe from both (32) and (33) that the long-run fall in consumption *exceeds* the rise in government expenditure. In other words, permanent government expenditure shocks "crowd out" private consumption by more than one-for-one in the long run.

We can easily explain this result using Figure 3, in which the increase in government spending g causes a downward shift in the $\dot{a} = 0$ locus, equal to Δg. This reflects the fact that lump-sum taxes τ must be increased one-for-one with g in order to maintain domestic fiscal balance. While the pre-tax real wage $\tilde{w} = \tilde{w}(r^*, B)$ is independent of changes in domestic fiscal policy, the after-tax real wage $\tilde{w} - \tau$ declines by Δg units, i.e., $\Delta(\tilde{w} - \tau) = -\Delta\tau = -\Delta g$.

Because the decline in initial consumption—as illustrated in Figure 3 by the jump from point G to point H—*falls short* of the reduction in the after-tax real wage, the private sector must dissave. This initial fall in consumption causes the symmetric MRS of status for consumption to decline, i.e., status loses in importance relative to consumption. Consequently, the effective rate of return r^e on saving falls below its steady-state value ρ. During the ensuing transition to the new steady state, private agents run down their stock of net assets and continue to reduce consumption, i.e., $\dot{a} < 0$ and $\dot{c} < 0$. As the economy moves along the new stable arm from point H to point I, the effective domestic interest rate then increases, returning to its unchanged steady-state value ρ.[25] To summarize, it is the combination of the fall in after-tax real wage income and the long-run reduction in net interest income that causes consumption to decline in the steady state by more than the increase in government expenditure. This "excessive" reduction in consumption is also clear in Figure 3 in which the fall in steady-state consumption from $\tilde{c}$ to $\tilde{c}'$ exceeds the vertical shift in the $\dot{a} = 0$ locus.[26]

[25] Recall from (24), that $\operatorname{sgn}(\dot{r}^e) = -\operatorname{sgn}(\dot{a})$ holds along the stable arm.

[26] For the Uzawa model, it is straightforward to show that an increase in g raises $\tilde{a}$ and leaves $\tilde{c}$ unchanged. On the other hand, in the upward-sloping debt framework, a rise in g causes a one-for-one decline in $\tilde{c}$, with $\tilde{a}$ left unaffected. In our view, the dynamic response of the economy under our specification is more general, because the long-run adjustment falls on both consumption and net assets. Analogous results hold for the productivity shock discussed in the next subsection.

3.2 Productivity Shocks

We will next consider the impact of a permanent increase in total factor productivity B on the steady-state values of net asset holdings and consumption. In contrast to fiscal shocks, changes in B will affect the pre-tax real wage $\tilde{w}$, $\partial \tilde{w}/\partial B = f(\tilde{k}) > 0$ [see (9)], while leaving lump-sum tax payments τ unchanged. Differentiating equations (31) with respect to B, we obtain

$$\frac{\partial \tilde{a}}{\partial B} = \frac{r_c^e(\tilde{c}, \tilde{a})}{\Psi} \frac{\partial \tilde{w}}{\partial B} > 0, \quad \frac{\partial \tilde{c}}{\partial B} = \frac{-r_a^e(\tilde{c}, \tilde{a})}{\Psi} \frac{\partial \tilde{w}}{\partial B} = \left[1 + \frac{r^* r_c^e(\tilde{c}, \tilde{a})}{\Psi}\right] \frac{\partial \tilde{w}}{\partial B} > \frac{\partial \tilde{w}}{\partial B}. \tag{34}$$

These relationships state that an improvement in productivity raises both the steady-state stock of assets and the level of consumption. Observe that the rise in steady-state consumption exceeds that of real wage income, which is due to the fact that steady-state interest income also increases. If, instead, preferences assume our modified Futagami-Shibata form, then the multipliers for $\tilde{a}$ and $\tilde{c}$ become:

$$\frac{\partial \tilde{a}}{\partial B} = \frac{\xi/\eta}{\rho - [1 + (\xi/\eta)]\, r^*} \frac{\partial \tilde{w}}{\partial B} > 0, \quad \frac{\partial \tilde{c}}{\partial B} = \frac{\rho - r^*}{\rho - [1 + (\xi/\eta)]\, r^*} \frac{\partial \tilde{w}}{\partial B} > \frac{\partial \tilde{w}}{\partial B}. \tag{35}$$

Using Figure 4, it is straightforward to explain these results. The improvement in productivity causes an upward shift in the $\dot{a} = 0$ locus, equal to the change in the pre-tax real wage: $\Delta \tilde{w} = (\partial \tilde{w}/\partial B)\Delta B = f(\tilde{k})\Delta B$. This shift in the $\dot{a} = 0$ locus leads to a new intersection with the $\dot{c} = 0$ locus at point P and results in a new steady state with higher values of net assets and consumption, $\tilde{a}'$ and $\tilde{c}'$. Nevertheless, the initial increase in consumption from point M to point N in Figure 4 is *less* than the increase in real wage income, which means that the private sector devotes part of this gain in real resources to asset accumulation. The initial rise in consumption is also reflected in the corresponding increase in the symmetric MRS of status for consumption. Because status gains in importance compared to consumption in the case of a permanent increase in B, the effective rate of return r^e jumps above its given steady-state value ρ. As the economy proceeds along the new stable arm between points N to point P in Figure 4, the private sector accumulates net assets, with consumption, consistent with the Euler equation (13), continuing to rise, i.e., $\dot{a} > 0$ and $\dot{c} > 0$. Due to the saddlepath relationship $\operatorname{sgn}(\dot{r}^e) = -\operatorname{sgn}(\dot{a})$, the effective rate of return declines, converging to its long run value ρ. As in the previous case of a fiscal expansion, the long-run increase in consumption is the sum of two effects: i) the immediate rise in after-tax real wage income due to the increase in B; ii) the gain in net interest income that is a consequence of the economy's accumulation of net assets. This result is also depicted in Figure 4, since the increase in steady-state consumption from $\tilde{c}$ to $\tilde{c}'$ is greater than the upward shift in the $\dot{a} = 0$ locus.[27]

4 Conclusions

In this paper we studied the implications of modifying the standard version of the small open economy Ramsey model by introducing preferences that depend on status as well as on own consumption. Following the branch of the macroeconomic literature that identifies status with relative wealth, we specified that status depends on the comparison between own and average holdings of net assets. What conclusions were we able to draw from our open economy model of status preference? First, we showed that our model eliminates one counter-intuitive property of the standard open economy Ramsey framework: that an impatient economy—in the sense that its pure rate of time preference exceeds the world interest rate—mortgages over time all its human and nonhuman wealth. Our economy, in contrast, potentially possesses an interior long-run equilibrium and saddlepath dynamics for consumption and net assets. The key variable that generates economic dynamics in our framework is the domestic effective rate of return, which, in addition to the world interest rate, depends on the private sector's willingness to substitute net assets for consumption. Next, using both general and parameterized specifications of the instantaneous utility function, we analyzed how status preference affects the short- and long-run properties of the open economy. Among our notable results, we found that the "importance" of asset accumulation for status seeking—defined in the parameterized specification—is crucial in determining the steady-state values of consumption and net assets. This, in turn, implies that even the direction of the economy's adjustment during the transition to the steady

[27] Additional comparative statics exercises for this model are, of course, possible. It is, for example, straightforward to show that while an increase in the pure rate of time preference ρ lowers both $\tilde{c}$ and $\tilde{a}$, an increase in the aversion to indebtedness $\bar{a}$, raises both $\tilde{c}$ and $\tilde{a}$. The effects of a permanent increase in the world interest rate are, in contrast, significantly more complex. In particular, not only the quantitative, but also the qualitative response of $\tilde{c}$ and $\tilde{a}$ depend on the importance of status. Among other results, we are able to demonstrate that a sufficient, but not necessary, condition for a rise in r^* to increase both $\tilde{c}$ and $\tilde{a}$ is that the economy is an international net creditor, i.e., $(\tilde{a} - b_0 - \tilde{k}) > 0$. This will be the case if the quest for status is sufficiently strong.

state is critically influenced by how "strong" is the private sector's status motive. We finally analyzed the role of status preference in determining the adjustment of the economy to macroeconomic shocks. This was illustrated by considering government expenditure and total factor productivity disturbances. We found that a permanent increase in government expenditure, financed by a rise in lump-sum taxes, "crowds out" private consumption by more than one-for-one. This is due to the fact that in response to this shift in fiscal policy, the private sector dissaves during the transition to the steady state. The resulting loss of net interest income augments the drop in after-tax real wage income due to higher lump-sum taxes and results in the "excessive" crowding out of private consumption. Analogously, a permanent improvement in total factor productivity leads to a long-run increase in private consumption that is greater than the rise in after-tax real wage income. This is caused by the accumulation of net assets during the transition to the steady state that, consequently, also raises net interest income.

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Figure 1: Phase Diagram

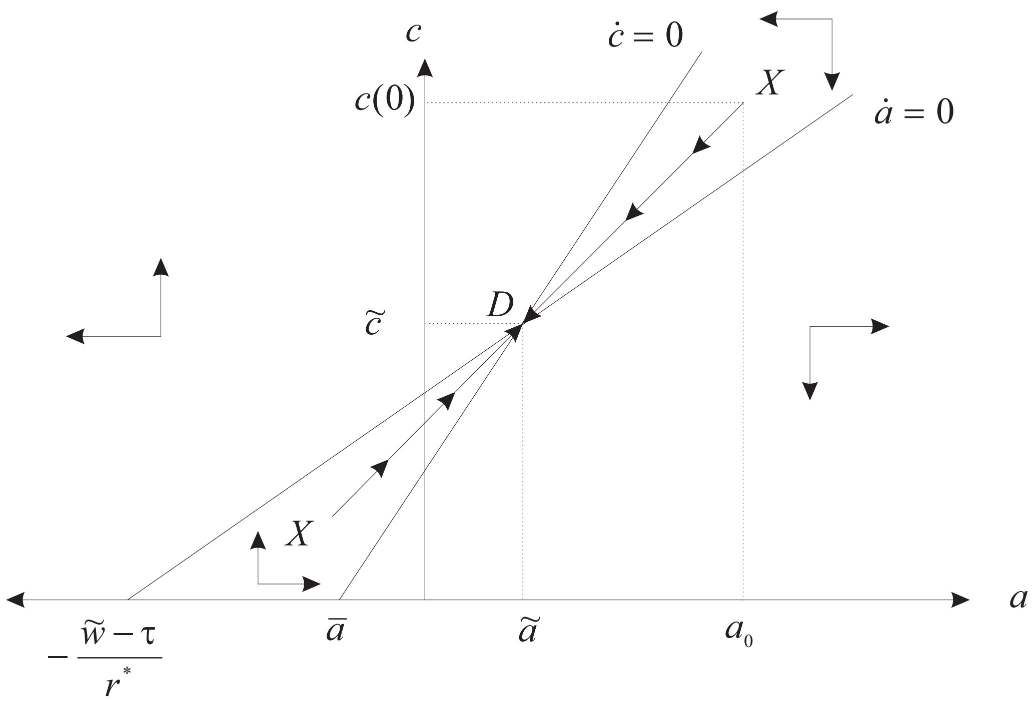


Figure 2: The Influence of Status

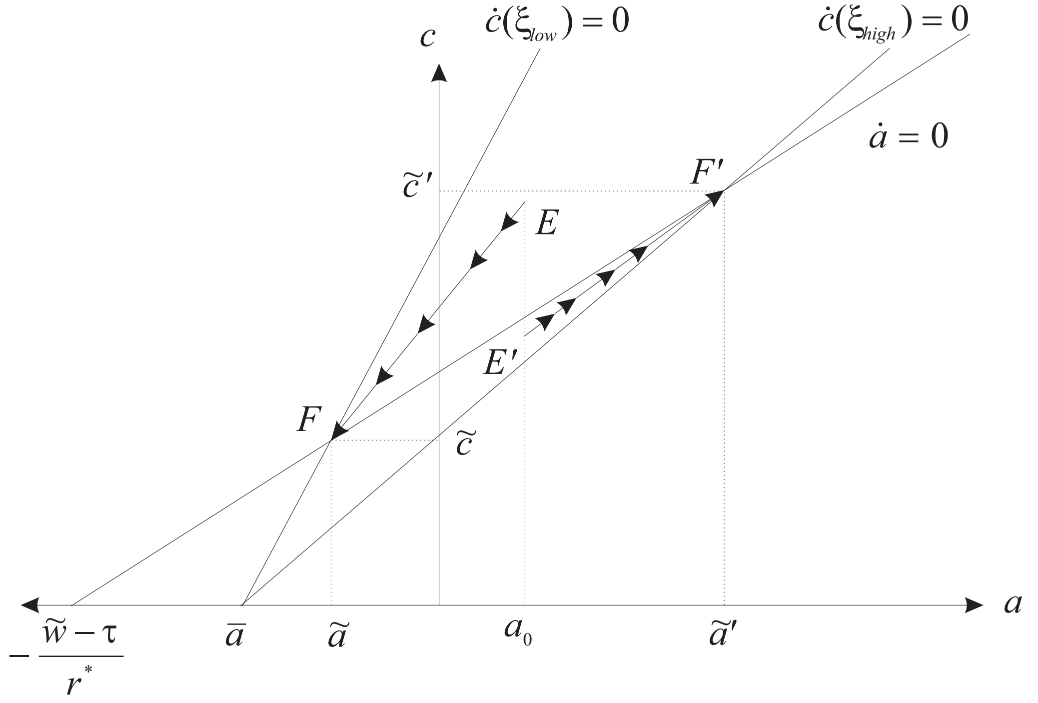


Figure 3: Fiscal Expansion

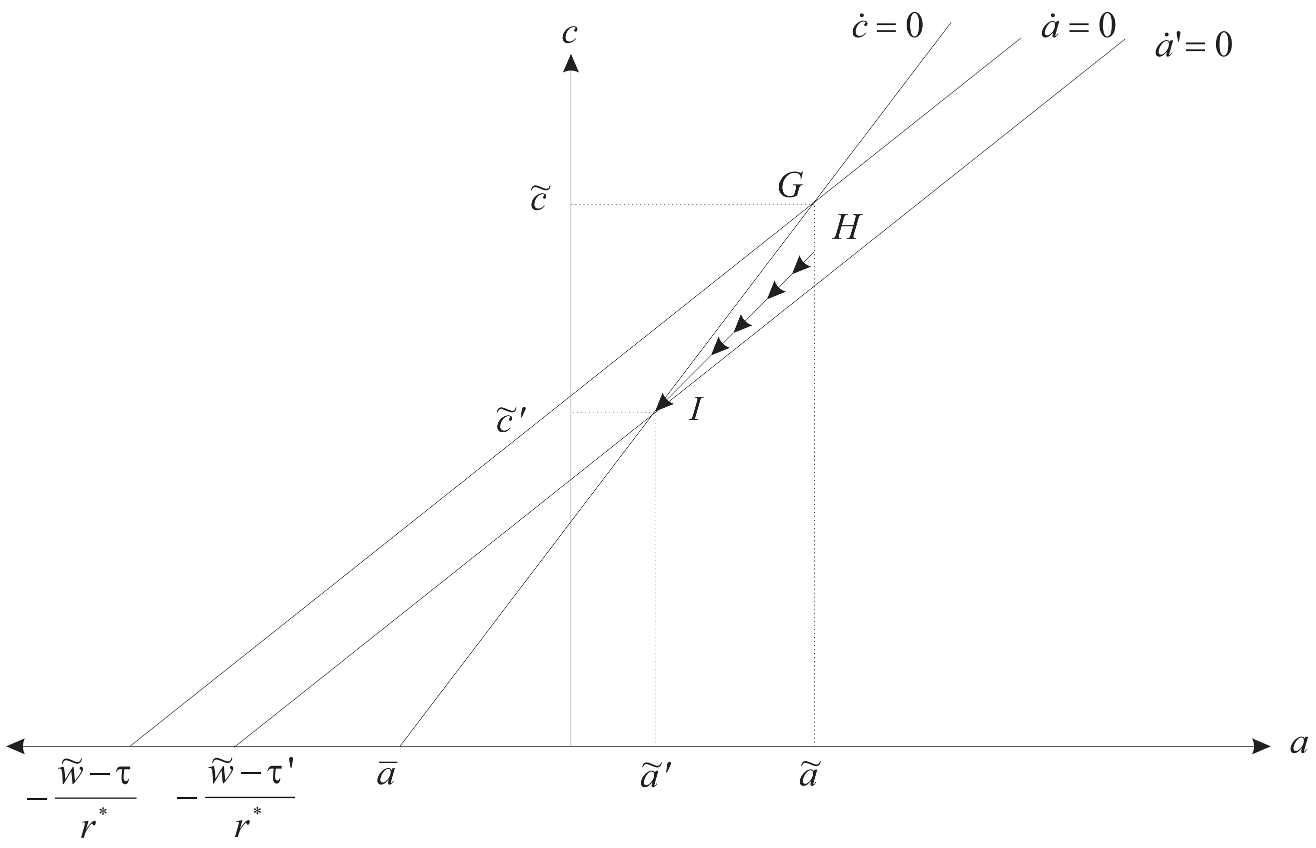
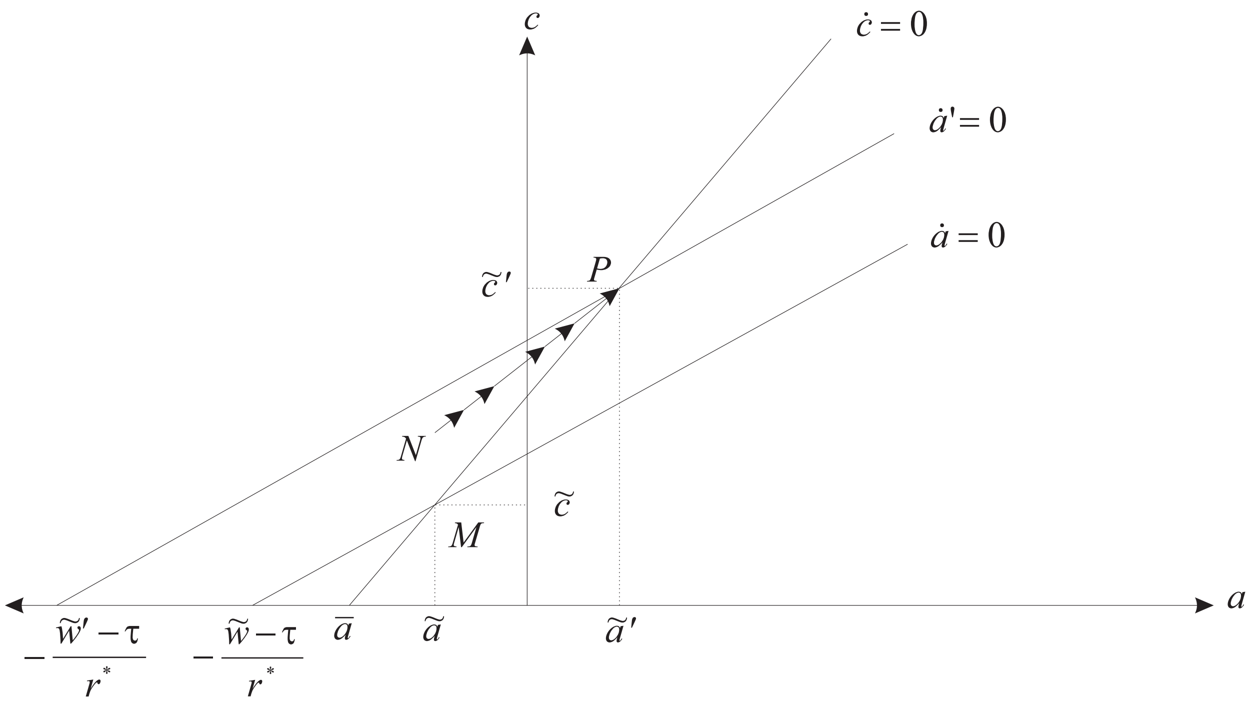


Figure 4: Productivity Shock



5 Mathematical Appendix

5.1 Properties of the status function (5)

On page 6 we state that the status function (5),

$$s(a, A) \equiv \varphi\left(\frac{a-\bar{a}}{A-\bar{a}}\right), \qquad \bar{a}<0, \quad \varphi'>0, \qquad \varphi''\leq 0,$$

which is defined for all $(a, A) \in (\bar{a}, \infty) \times (\bar{a}, \infty)$, satisfies all properties given in (4), $s_a > 0$, $s_A < 0$, $s_{aa} \leq 0$, for all $(a, A) \in (\bar{a}, \infty) \times (\bar{a}, \infty)$. From

$$s_a\left(a, A\right) = \varphi'\left(\frac{a-\bar{a}}{A-\bar{a}}\right)\frac{1}{A-\bar{a}} > 0, \tag{36}$$

$$s_{aa}\left(a, A\right) = \varphi''\left(\frac{a-\bar{a}}{A-\bar{a}}\right)\frac{1}{\left(A-\bar{a}\right)^2} \leq 0, \tag{37}$$

$$s_A(a, A) = -\varphi'\left(\frac{a-\bar{a}}{A-\bar{a}}\right)\frac{a-\bar{a}}{\left(A-\bar{a}\right)^2} < 0, \tag{38}$$

it is obvious that our statement is true.

5.2 The properties of the current-value Hamiltonian

On page 7 we state that due to the assumptions made in (1) and (5), the current-value Hamiltonian,

$$H(c, a, \lambda) = U\left(c, \varphi\left(\frac{a-\bar{a}}{A-\bar{a}}\right)\right) + \lambda(r^*a + w - \tau - c), \tag{39}$$

is jointly concave in the control variable c and the state variable a. We also state that in this case the necessary conditions for an interior optimum, $H_c = 0$ and $\dot{\lambda} = \rho\lambda - H_a$, (7) and (8), are sufficient for optimality, as long as the limiting transversality condition $\lim_{t\to\infty} \lambda a e^{-\rho t} = 0$ holds.

The first statement can be verified as follows: taking derivatives of (39) we obtain:

$$H_c = U_c\left(c, \varphi\left(\frac{a-\bar{a}}{A-\bar{a}}\right)\right) - \lambda,$$

$$H_{cc} = U_{cc}\left(c, \varphi\left(\frac{a-\bar{a}}{A-\bar{a}}\right)\right),$$

$$H_{ca} = U_{cs}\left(c, \varphi\left(\frac{a-\bar{a}}{A-\bar{a}}\right)\right)\varphi'\left(\frac{a-\bar{a}}{A-\bar{a}}\right)\frac{1}{A-\bar{a}},$$

$$H_a = U_s\left(c, \varphi\left(\frac{a-\bar{a}}{A-\bar{a}}\right)\right)\varphi'\left(\frac{a-\bar{a}}{A-\bar{a}}\right)\frac{1}{A-\bar{a}} + \lambda r^*,$$

$$H_{aa} = \frac{1}{(A-\bar{a})^2} \left\{ U_{ss}\left(c, \varphi\left(\frac{a-\bar{a}}{A-\bar{a}}\right)\right) \left[\varphi'\left(\frac{a-\bar{a}}{A-\bar{a}}\right)\right]^2 + U_s\left(c, \varphi\left(\frac{a-\bar{a}}{A-\bar{a}}\right)\right) \varphi''\left(\frac{a-\bar{a}}{A-\bar{a}}\right) \right\}.$$

Using a more compact notation, we have

$$H_{cc} = U_{cc}, \qquad H_{ca} = \frac{1}{A-\bar{a}} U_{cs}\varphi', \qquad H_{aa} = \frac{1}{(A-\bar{a})^2}\left[U_{ss}(\varphi')^2 + U_s\varphi''\right].$$

From these results, it follows that

$$H_{cc}H_{aa} - H_{ca}^2 = \frac{1}{(A-\bar{a})^2}\left[\left(U_{cc}U_{ss} - U_{cs}^2\right)(\varphi')^2 + U_{cc}U_s\varphi''\right].$$

The assumptions made in (1), $U_c > 0$, $U_s > 0$, $U_{cc} < 0$, $U_{ss} \leq 0$, $U_{cc}U_{ss} - U_{cs}^2 \geq 0$, and in (5), $\varphi' > 0$, $\varphi'' \leq 0$, ensure that

$$H_{cc} < 0, \qquad H_{aa} \leq 0, \qquad H_{cc}H_{aa} - H_{ca}^2 \geq 0. \tag{40}$$

Consequently, the Hamiltonian is jointly concave in c and a. This, in turn, implies that the maximized Hamiltonian $H^{\max}$ is concave in the state variable a (although this is clear, a proof will be given below). As is well-known from the theory of optimal control, this property of the maximized Hamiltonian $H^{\max}$ ensures that as long as the limiting transversality condition $\lim_{t\to\infty} \lambda a e^{-\rho t} = 0$ holds, the necessary conditions for an interior optimum, $H_c = 0$ and $\dot{\lambda} = \rho\lambda - H_a$, (7) and (8), are sufficient for optimality.

In the rest of this subsection we will show that if the Hamiltonian satisfies (40), then the maximized Hamiltonian $H^{\max}$ is concave in a. First, the necessary optimality condition $H_c(c, a, \lambda) = 0$ can be solved for c in the form $c = \hat{c}(a, \lambda)$, where

$$\hat{c}_a(a, \lambda) = -\frac{H_{ca}(\hat{c}(a, \lambda), a, \lambda)}{H_{cc}(\hat{c}(a, \lambda), a, \lambda)}. \tag{41}$$

The maximized Hamiltonian is given by $H^{\max} = H^{\max}(a, \lambda) \equiv H(\hat{c}(a, \lambda), a, \lambda)$. Invoking the envelope theorem we have

$$H_a^{\max}(a, \lambda) = H_a(\hat{c}(a, \lambda), a, \lambda).$$

Differentiating once more with respect to a we obtain:

$$H_{aa}^{\max}(a, \lambda) = H_{ac}(\hat{c}(a, \lambda), a, \lambda)\hat{c}_a(a, \lambda) + H_{aa}(\hat{c}(a, \lambda), a, \lambda).$$

Substitution of (41) and rearranging yields

$$H_{aa}^{\max}(a, \lambda) = \frac{H_{cc}(\hat{c}(a, \lambda), a, \lambda)H_{aa}(\hat{c}(a, \lambda), a, \lambda) - [H_{ca}(\hat{c}(a, \lambda), a, \lambda)]^2}{H_{cc}(\hat{c}(a, \lambda), a, \lambda)}.$$

From (40) it follows that $H_{aa}^{\max} \leq 0$, i.e., the maximized Hamiltonian is concave in the state variable a.

5.3 The properties of $\tilde{k} = \tilde{k}(r^*, B)$ and $\tilde{w} = \tilde{w}(r^*, B)$ [equation (9)]

The steady-state values of $\tilde{k}$ and $\tilde{w}$ are implicitly defined by

$$r^* = Bf'\left(\tilde{k}\right), \qquad \tilde{w} = B\left[f\left(\tilde{k}\right) - \tilde{k}f'\left(\tilde{k}\right)\right]. \tag{42}$$

It is clear that $\tilde{k} = \tilde{k}(r^*, B)$ and $\tilde{w} = \tilde{w}(r^*, B)$. First, implicit differentiation of (42) with respect to r^* yields

$$1 = Bf''\left(\tilde{k}\right)\tilde{k}_{r^*}, \qquad \tilde{w}_{r^*} = -B\tilde{k}f''\left(\tilde{k}\right)\tilde{k}_{r^*}.$$

Solving for $\tilde{k}_{r^*}$ and $\tilde{w}_{r^*}$ we calculate

$$\tilde{k}_{r^*} = [Bf''(\tilde{k})]^{-1}, \qquad \tilde{w}_{r^*} = -\tilde{k} < 0.$$

Second, implicit differentiation of (42) with respect to B yields

$$0 = f'\left(\tilde{k}\right) + Bf''\left(\tilde{k}\right)\tilde{k}_B, \qquad \tilde{w}_B = f\left(\tilde{k}\right) - \tilde{k}f'\left(\tilde{k}\right) - B\tilde{k}f''\left(\tilde{k}\right)\tilde{k}_B.$$

Solving for $\tilde{k}_B$ and $\tilde{w}_B$ we obtain

$$\tilde{k}_B = -f'\left(\tilde{k}\right)\left[Bf''\left(\tilde{k}\right)\right]^{-1}, \qquad \tilde{w}_B = f\left(\tilde{k}\right).$$

5.4 The derivation of the modified Euler equation (13)–(14)

Differentiation of (10), $U_c(c, \varphi(1)) = \lambda$, with respect to time t yields

$$U_{cc}(c, \varphi(1))\dot{c} = \dot{\lambda}. \tag{43}$$

Substituting (10) and (43) into (11),

$$\dot{\lambda} = (\rho - r^*)\lambda - U_s(c, \varphi(1))\frac{\varphi'(1)}{a - \bar{a}},$$

we obtain

$$\dot{c} = -\frac{1}{U_{cc}(c, \varphi(1))}\left[(r^* - \rho)U_c(c, \varphi(1)) + U_s(c, \varphi(1))\frac{\varphi'(1)}{a - \bar{a}}\right].$$

In order to allow for immediate comparisons with the standard Euler equation for consumption, it will be convenient to rewrite this differential equation as

$$\dot{c} = c\left[-\frac{U_c(c, \varphi(1))}{cU_{cc}(c, \varphi(1))}\right]\left[r^* + \frac{U_s(c, \varphi(1))}{U_c(c, \varphi(1))}\frac{\varphi'(1)}{a - \bar{a}} - \rho\right].$$

Introducing the definitions of the *effective* elasticity of intertemporal substitution σ^e and the *effective* domestic, or internal, rate of return on assets r^e, respectively, given by (14),

$$\sigma^e(c) \equiv -\frac{U_c(c,\varphi(1))}{cU_{cc}(c,\varphi(1))}, \qquad r^e(r^*,c,a) \equiv r^* + \frac{U_s(c,\varphi(1))}{U_c(c,\varphi(1))}\frac{\varphi'(1)}{a-\bar{a}},$$

we obtain the modified Euler equation (13), $\dot{c} = c\sigma^e(c)[r^e(r^*,c,a)-\rho]$.

5.5 The symmetric MRS

On page 9 we state that

$$\frac{U_s(c,\varphi(1))}{U_c(c,\varphi(1))}\frac{\varphi'(1)}{a-\bar{a}} = \frac{U_s(c,\varphi(1))}{U_c(c,\varphi(1))}s_a\left(a,a\right) = \frac{U_s(c,s(a,A))}{U_c(c,s(a,A))}s_a(a,A)\bigg|_{a=A}.$$

The first equality follows from

$$s_a\left(a,a\right) = \frac{\varphi'(1)}{a-\bar{a}}, \tag{44}$$

which is easily verified by evaluating (36) at $(a,A)=(a,a)$:

$$s_a\left(a,a\right) = \varphi'\left(\frac{a-\bar{a}}{a-\bar{a}}\right)\frac{1}{a-\bar{a}} = \frac{\varphi'(1)}{a-\bar{a}}.$$

The second equality then follows from

$$s(a,a) = \varphi\left(\frac{a-\bar{a}}{a-\bar{a}}\right) = \varphi\left(1\right). \tag{45}$$

5.6 The properties of $s_a\left(a,a\right)$

On page 10 we state that

$$\frac{ds_a(a,a)}{da} = s_{aa}(a,a) + s_{aA}(a,a) = -\frac{1}{(a-\bar{a})^2}\varphi'(1) < 0. \tag{46}$$

In the following, we will give two proofs, a very simple and straightforward one and a second one that yields the additional information why the second derivative of the function φ does not play any role in a symmetric states.

Proof #1: Taking the total derivative of (44) with respect to a we obtain (46) directly.

Proof #2: Differentiation of (36) with respect to A yields

$$s_{aA}\left(a,A\right) = -\frac{1}{(A-\bar{a})^2}\left[\varphi''\left(\frac{a-\bar{a}}{A-\bar{a}}\right)\frac{a-\bar{a}}{A-\bar{a}} + \varphi'\left(\frac{a-\bar{a}}{A-\bar{a}}\right)\right]. \tag{47}$$

Evaluating both (37) and (47) at $(a, A) = (a, a)$, we obtain

$$s_{aa}(a,a) = \frac{1}{(a-\bar{a})^2}\varphi''(1), \tag{48}$$

$$s_{aA}(a,a) = -\frac{1}{(a-\bar{a})^2}\left[\varphi''(1) + \varphi'(1)\right]. \tag{49}$$

From (48) and (49), it follows that (46) holds. Note that the expressions which include φ'' cancel out.

5.7 The transversality condition of our model

On page 10 we state that the transversality condition of our model can be written as

$$\lim_{t\to\infty}\left\{a(t)\exp\left[-\int_0^t r^e(r^*, c(v), a(v))\,dv\right]\right\} = 0. \tag{50}$$

Proof: Equation (11),

$$\dot{\lambda} = (\rho - r^*)\,\lambda - U_s(c, \varphi(1))\frac{\varphi'(1)}{a-\bar{a}},$$

can be rewritten as

$$\dot{\lambda} = (\rho - r^*)\,\lambda - \left[\frac{U_s(c,\varphi(1))}{U_c(c,\varphi(1))}\frac{\varphi'(1)}{a-\bar{a}}\right]U_c(c,\varphi(1)). \tag{51}$$

Using (10), $U_c(c, \varphi(1)) = \lambda$, (51) can be rewritten as

$$\dot{\lambda} = \left[\rho - \left(r^* + \frac{U_s(c,\varphi(1))}{U_c(c,\varphi(1))}\frac{\varphi'(1)}{a-\bar{a}}\right)\right]\lambda. \tag{52}$$

Using definition of the effective rate of return r^e given in (14),

$$r^e(r^*, c, a) \equiv r^* + \frac{U_s(c,\varphi(1))}{U_c(c,\varphi(1))}\frac{\varphi'(1)}{a-\bar{a}},$$

the differential equation (52) can also be written as

$$\dot{\lambda} = -\left[r^e(r^*, c, a) - \rho\right]\lambda. \tag{53}$$

Integration of (53) yields

$$\lambda(t) = \lambda(0)\,e^{\rho t}\exp\left[-\int_0^t r^e(r^*, c(v), a(v))\,dv\right]. \tag{54}$$

Since $\lambda(0) = U_c(c(0), \varphi(1)) > 0$ due to the assumption that $U_c > 0$, the transversality condition given on page 7, $\lim_{t\to\infty} e^{-\rho t}\lambda a = 0$, holds if and only if the condition (50) is satisfied.

5.8 The derivation of the pair of differential equations (19)

The dynamic evolution of a and c over time is governed by the pair of differential equations (12) and (13):

$$\dot{a} = r^* a + \tilde{w} - (g + r^* b_0) - c, \tag{55}$$

$$\dot{c} = c\sigma^e(c)[r^e(r^*, c, a) - \rho]. \tag{56}$$

In general, linearizing a pair of differential equations $\dot{a} = \dot{a}\,(a, c)$ and $\dot{c} = \dot{c}\,(a, c)$ about a steady state $(\tilde{a}, \tilde{c})$, in which $\dot{a}(\tilde{a}, \tilde{c}) = \dot{c}(\tilde{a}, \tilde{c}) = 0$ holds, yields

$$\dot{a}\,(a, c) = \frac{\partial \dot{a}}{\partial a}(\tilde{a}, \tilde{c})\,(a - \tilde{a}) + \frac{\partial \dot{a}}{\partial c}(\tilde{a}, \tilde{c})\,(c - \tilde{c})\,, \tag{57}$$

$$\dot{c}\,(a, c) = \frac{\partial \dot{c}}{\partial a}(\tilde{a}, \tilde{c})\,(a - \tilde{a}) + \frac{\partial \dot{c}}{\partial c}(\tilde{a}, \tilde{c})\,(c - \tilde{c})\,. \tag{58}$$

Differentiating (55) and (56) with respect to a and c we obtain:

$$\frac{\partial \dot{a}}{\partial a} = r^*, \quad \frac{\partial \dot{a}}{\partial c} = -1, \tag{59}$$

$$\frac{\partial \dot{c}}{\partial a} = c\sigma^e\,(c)\,r^e_a\,(c, a)\,, \quad \frac{\partial \dot{c}}{\partial c} = \frac{d\,[c\sigma^e\,(c)]}{dc}\,[r^e\,(r^*, c, a) - \rho] + c\sigma^e\,(c)\,r^e_c\,(c, a)\,. \tag{60}$$

Evaluation of (60) at the steady state $(\tilde{a}, \tilde{c})$, in which $r^e\,(r^*, \tilde{c}, \tilde{a}) = \rho$ holds, yields

$$\frac{\partial \dot{c}}{\partial a}(\tilde{a}, \tilde{c}) = \tilde{c}\sigma^e\,(\tilde{c})\,r^e_a\,(\tilde{c}, \tilde{a})\,, \qquad \frac{\partial \dot{c}}{\partial c} = \tilde{c}\sigma^e\,(\tilde{c})\,r^e_c\,(\tilde{c}, \tilde{a})\,. \tag{61}$$

Using (57), (58), (59), and (61), it follows that under the linear approximation the dynamic evolution of a and c is governed by (19),

$$\begin{pmatrix} \dot{a} \\ \dot{c} \end{pmatrix} = \begin{pmatrix} r^* & -1 \\ \tilde{c}\sigma^e(\tilde{c})r^e_a(\tilde{c}, \tilde{a}) & \tilde{c}\sigma^e(\tilde{c})r^e_c(\tilde{c}, \tilde{a}) \end{pmatrix} \begin{pmatrix} a - \tilde{a} \\ c - \tilde{c} \end{pmatrix}.$$

5.9 Footnote 15 – Proof

In footnote 15 we state that

$$[\mathrm{tr}\,(\mathbf{J})]^2 - 4\det(\mathbf{J}) = [r^* - \tilde{c}\sigma^e(\tilde{c})r^e_c(\tilde{c}, \tilde{a})]^2 - 4\tilde{c}\sigma^e(\tilde{c})r^e_a(\tilde{c}, \tilde{a}).$$

Proof: Using (22),

$$\mathrm{tr}\;(\mathbf{J}) = r^* + \tilde{c}\sigma^e(\tilde{c})r^e_c(\tilde{c}, \tilde{a}) > 0, \qquad \det(\mathbf{J}) = \tilde{c}\sigma^e(\tilde{c})[r^* r^e_c(\tilde{c}, \tilde{a}) + r^e_a(\tilde{c}, \tilde{a})],$$

we obtain

$$
\begin{aligned}
& [\operatorname{tr}(\mathbf{J})]^2 - 4\det(\mathbf{J}) \\
= \;& [r^* + \tilde{c}\sigma^e(\tilde{c})\, r_c^e(\tilde{c},\tilde{a})]^2 - 4\tilde{c}\sigma^e(\tilde{c})\,[r^* r_c^e(\tilde{c},\tilde{a}) + r_a^e(\tilde{c},\tilde{a})] \\
= \;& (r^*)^2 + 2r^*\tilde{c}\sigma^e(\tilde{c})\, r_c^e(\tilde{c},\tilde{a}) + [\tilde{c}\sigma^e(\tilde{c})\, r_c^e(\tilde{c},\tilde{a})]^2 - 4\tilde{c}\sigma^e(\tilde{c})\, r^* r_c^e(\tilde{c},\tilde{a}) \\
& -4\tilde{c}\sigma^e(\tilde{c})\, r_a^e(\tilde{c},\tilde{a}) \\
= \;& (r^*)^2 - 2\tilde{c}\sigma^e(\tilde{c})\, r^* r_c^e(\tilde{c},\tilde{a}) + [\tilde{c}\sigma^e(\tilde{c})\, r_c^e(\tilde{c},\tilde{a})]^2 - 4\tilde{c}\sigma^e(\tilde{c})\, r_a^e(\tilde{c},\tilde{a}) \\
= \;& [r^* - \tilde{c}\sigma^e(\tilde{c})\, r_c^e(\tilde{c},\tilde{a})]^2 - 4\tilde{c}\sigma^e(\tilde{c})\, r_a^e(\tilde{c},\tilde{a}).
\end{aligned}
$$

5.10 The solution to (19) and the stable arm (23)

In the following we will restrict attention to the case in which the steady state $(\tilde{a},\tilde{c})$ is a saddlepoint so that $\mu_1 < 0$ and $\mu_2 > 0$ holds. Eigenvectors corresponding to the roots μ_1 and μ_2 are given by

$$
\begin{pmatrix} 1 \\ r^* - \mu_1 \end{pmatrix} \quad \text{and} \quad \begin{pmatrix} 1 \\ r^* - \mu_2 \end{pmatrix},
$$

respectively. Hence, the general solution to (19) takes the following form:

$$
\begin{pmatrix} a \\ c \end{pmatrix} = \begin{pmatrix} \tilde{a} \\ \tilde{c} \end{pmatrix} + D_1 \begin{pmatrix} 1 \\ r^* - \mu_1 \end{pmatrix} e^{\mu_1 t} + D_2 \begin{pmatrix} 1 \\ r^* - \mu_2 \end{pmatrix} e^{\mu_2 t}.
$$

where D_1 and D_2 are constants to be determined. Employing the initial condition and the transversality condition we obtain $D_1 = a_0 - \tilde{a}$ and $D_2 = 0$. Hence, in the linearized model the solution is given by

$$
\begin{pmatrix} a \\ c \end{pmatrix} = \begin{pmatrix} \tilde{a} \\ \tilde{c} \end{pmatrix} + (a_0 - \tilde{a}) \begin{pmatrix} 1 \\ r^* - \mu_1 \end{pmatrix} e^{\mu_1 t}.
$$

Using (20),

$$
(r^* - \mu)[\tilde{c}\sigma^e(\tilde{c}) r_c^e(\tilde{c},\tilde{a}) - \mu] + \tilde{c}\sigma^e(\tilde{c}) r_a^e(\tilde{c},\tilde{a}) = 0,
$$

this solution can also be written as

$$
\begin{pmatrix} a \\ c \end{pmatrix} = \begin{pmatrix} \tilde{a} \\ \tilde{c} \end{pmatrix} + (a_0 - \tilde{a}) \begin{pmatrix} 1 \\ -\frac{\tilde{c}\sigma^e(\tilde{c}) r_a^e(\tilde{c},\tilde{a})}{\tilde{c}\sigma^e(\tilde{c}) r_c^e(\tilde{c},\tilde{a}) - \mu_1} \end{pmatrix} e^{\mu_1 t}.
$$

From these two alternative representations of the solution, it follows that there are two alternative representations of the stable arm given, i.e., those given in (23):

$$c - \tilde{c} = (r^* - \mu_1)(a - \tilde{a}) = -\frac{\tilde{c}\sigma^e(\tilde{c}) r_a^e(\tilde{c}, \tilde{a})}{\tilde{c}\sigma^e(\tilde{c}) r_c^e(\tilde{c}, \tilde{a}) - \mu_1}(a - \tilde{a}).$$

5.11 Proof of (24)

In this subsection we will verify the validity of equation (24),

$$r^e(r^*, c, a) - \rho = \frac{(r^* - \mu_1)\mu_1}{\tilde{c}\sigma^e(\tilde{c})}(a - \tilde{a}) = -\frac{\mu_1 r_a^e(\tilde{c}, \tilde{a})}{\tilde{c}\sigma^e(\tilde{c}) r_c^e(\tilde{c}, \tilde{a}) - \mu_1}(a - \tilde{a}), \tag{62}$$

which describes the co-movement of r^e and a along the stable arm.

Proof: A linear approximation of $r^e(r^*, c, a)$ about a steady state $(\tilde{c}, \tilde{a})$, in which $r^e(r^*, \tilde{c}, \tilde{a}) = \rho$ holds, yields

$$r^e(r^*, c, a) - \rho = r_c^e(\tilde{c}, \tilde{a})(c - \tilde{c}) + r_a^e(\tilde{c}, \tilde{a})(a - \tilde{a}). \tag{63}$$

First, substitution of the first equality in (23), $c - \tilde{c} = (r^* - \mu_1)(a - \tilde{a})$, into (63) yields

$$r^e(r^*, c, a) - \rho = \left[r_c^e(\tilde{c}, \tilde{a})(r^* - \mu_1) + r_a^e(\tilde{c}, \tilde{a})\right](a - \tilde{a}). \tag{64}$$

From (20), i.e., $0 = (r^* - \mu)[\tilde{c}\sigma^e(\tilde{c}) r_c^e(\tilde{c}, \tilde{a}) - \mu] + \tilde{c}\sigma^e(\tilde{c}) r_a^e(\tilde{c}, \tilde{a})$, it follows that

$$r_c^e(\tilde{c}, \tilde{a})(r^* - \mu_1) + r_a^e(\tilde{c}, \tilde{a}) = \frac{(r^* - \mu_1)\mu_1}{\tilde{c}\sigma^e(\tilde{c})}. \tag{65}$$

Substitution of (65) into (64) yields

$$r^e(r^*, c, a) - \rho = \frac{(r^* - \mu_1)\mu_1}{\tilde{c}\sigma^e(\tilde{c})}(a - \tilde{a}),$$

which equals the first equality in (24) [= (62)].

Substitution of the second equality in (23),

$$c - \tilde{c} = -\frac{\tilde{c}\sigma^e(\tilde{c}) r_a^e(\tilde{c}, \tilde{a})}{\tilde{c}\sigma^e(\tilde{c}) r_c^e(\tilde{c}, \tilde{a}) - \mu_1}(a - \tilde{a}),$$

into (63) yields

$$\begin{aligned} r^e(r^*, c, a) - \rho &= \left[-\frac{\tilde{c}\sigma^e(\tilde{c}) r_c^e(\tilde{c}, \tilde{a}) r_a^e(\tilde{c}, \tilde{a})}{\tilde{c}\sigma^e(\tilde{c}) r_c^e(\tilde{c}, \tilde{a}) - \mu_1} + r_a^e(\tilde{c}, \tilde{a})\right](a - \tilde{a}) \\ &= -\frac{\mu_1 r_a^e(\tilde{c}, \tilde{a})}{\tilde{c}\sigma^e(\tilde{c}) r_c^e(\tilde{c}, \tilde{a}) - \mu_1}(a - \tilde{a}). \end{aligned} \tag{66}$$

Obviously, (66) coincides with the second equality in (24) [= (62)].

5.12 The properties of the utility function (25)

If $U(c,s)$ takes the form given by (25),

$$U(c,s) = \frac{1}{1-\theta}\left[\left(c^{\eta}s^{\beta}\right)^{1-\theta} - 1\right],$$

then

$$U_c(c,s) = \eta c^{-[1-\eta(1-\theta)]}s^{\beta(1-\theta)}, \tag{67}$$

$$U_s(c,s) = \beta c^{\eta(1-\theta)}s^{-[1-\beta(1-\theta)]}, \tag{68}$$

$$U_{cc}(c,s) = -\left[1-\eta\left(1-\theta\right)\right]\eta c^{-[2-\eta(1-\theta)]}s^{\beta(1-\theta)}, \tag{69}$$

$$U_{ss}(c,s) = -\left[1-\beta\left(1-\theta\right)\right]\beta c^{\eta(1-\theta)}s^{-[2-\beta(1-\theta)]},$$

$$U_{cs}(c,s) = \eta\beta\left(1-\theta\right)c^{-[1-\eta(1-\theta)]}s^{-[1-\beta(1-\theta)]},$$

$$U_{cc}(c,s)U_{ss}(c,s) - \left[U_{cs}(c,s)\right]^2 = \left[1-\left(\beta+\eta\right)\left(1-\theta\right)\right]\eta\beta c^{-2[1-\eta(1-\theta)]}s^{-2[1-\beta(1-\theta)]}.$$

Our assumptions $\eta > 0$, $\beta > 0$, $\theta > 0$, $1-\eta(1-\theta) > 0$, $1-\beta(1-\theta) \geq 0$, and $1-(\beta+\eta)(1-\theta) \geq 0$ ensure that $U(c,s)$ satisfies all properties listed in (1), $U_c > 0$, $U_s > 0$, $U_{cc} < 0$, $U_{ss} \leq 0$, and $U_{cc}U_{ss} - U_{cs}^2 \geq 0$. Moreover, our assumptions ensure that both (2) and (3) hold:

$$U_{sc}U_c - U_sU_{cc} = \eta\beta c^{-2[1-\eta(1-\theta)]}s^{-[1-2\beta(1-\theta)]} > 0,$$

$$\lim_{c\to 0} U_c(c,s) = \lim_{c\to 0}\left(\frac{\eta s^{\beta(1-\theta)}}{c^{1-\eta(1-\theta)}}\right) = \infty, \qquad \lim_{c\to\infty} U_c(c,s) = \lim_{c\to\infty}\left(\frac{\eta s^{\beta(1-\theta)}}{c^{1-\eta(1-\theta)}}\right) = 0.$$

5.13 The solutions for σ^e and r^e in the parameterized model

Evaluating (67)–(69) at $(c,s) = (c,\varphi(1))$ and substituting the resulting expressions into the definitions of σ^e and r^e given in (14),

$$\sigma^e(c) \equiv -\frac{U_c(c,\varphi(1))}{cU_{cc}(c,\varphi(1))}, \qquad r^e(r^*,c,a) \equiv r^* + \frac{U_s(c,\varphi(1))}{U_c(c,\varphi(1))}\frac{\varphi'(1)}{a-\bar{a}},$$

we obtain

$$\sigma^e = -\frac{\eta c^{-[1-\eta(1-\theta)]}\left[\varphi\left(1\right)\right]^{\beta(1-\theta)}}{-c\left[1-\eta\left(1-\theta\right)\right]\eta c^{-[2-\eta(1-\theta)]}\left[\varphi\left(1\right)\right]^{\beta(1-\theta)}} = \frac{1}{1-\eta\left(1-\theta\right)} > 0, \tag{70}$$

$$r^e = r^* + \frac{\beta c^{\eta(1-\theta)}\left[\varphi\left(1\right)\right]^{-[1-\beta(1-\theta)]}}{\eta c^{-[1-\eta(1-\theta)]}\left[\varphi\left(1\right)\right]^{\beta(1-\theta)}}\frac{\varphi'(1)}{a-\bar{a}} = r^* + \frac{\beta\varphi'(1)}{\eta\varphi\left(1\right)}\frac{c}{a-\bar{a}}. \tag{71}$$

Introducing the definition $\xi \equiv [\beta\varphi'(1)]/\varphi(1)$, the solution for r^e can be written as

$$r^e = r^* + (\xi/\eta)\frac{c}{a-\bar{a}}, \tag{72}$$

which equals (26).

5.14 The solutions for $\mathrm{tr}\,(\mathbf{J})$ and $\det\,(\mathbf{J})$ in the parameterized model

From (72), it follows that the partial derivatives of r^e with respect to c and a are given by

$$r_c^e\,(c,a) = (\xi/\eta)\frac{1}{a-\bar{a}} > 0, \tag{73}$$

$$r_a^e\,(c,a) = -(\xi/\eta)\frac{c}{(a-\bar{a})^2} < 0. \tag{74}$$

Evaluating (73) and (74) at the steady-state values $(\tilde{a}, \tilde{c})$ [see (28) and (29)], which can be rewritten as

$$\tilde{a} = \bar{a} + \frac{\xi/\eta}{\rho - [1+(\xi/\eta)]r^*}\left[r^*\bar{a} + \tilde{w} - r^*b_0 - g\right], \tag{75}$$

$$\tilde{c} = \frac{\rho - r^*}{\rho - [1+(\xi/\eta)]r^*}\left[r^*\bar{a} + \tilde{w} - r^*b_0 - g\right], \tag{76}$$

we obtain:

$$r_c^e\,(\tilde{c},\tilde{a}) = \frac{\rho - [1+(\xi/\eta)]r^*}{r^*\bar{a} + \tilde{w} - r^*b_0 - g}, \tag{77}$$

$$r_a^e\,(\tilde{c},\tilde{a}) = -\frac{(\rho - r^*)\left\{\rho - [1+(\xi/\eta)]r^*\right\}}{(\xi/\eta)\left[r^*\bar{a} + \tilde{w} - r^*b_0 - g\right]}, \tag{78}$$

$$r^*r_c^e(\tilde{c},\tilde{a}) + r_a^e(\tilde{c},\tilde{a}) = -\frac{\left\{\rho - [1+(\xi/\eta)]r^*\right\}^2}{(\xi/\eta)\left[r^*\bar{a} + \tilde{w} - r^*b_0 - g\right]}. \tag{79}$$

Substitution of (70) and (76)–(79) into (22),

$$\mathrm{tr}\,(\mathbf{J}) = r^* + \tilde{c}\sigma^e(\tilde{c})r_c^e(\tilde{c},\tilde{a}), \qquad \det\,(\mathbf{J}) = \tilde{c}\sigma^e(\tilde{c})[r^*r_c^e(\tilde{c},\tilde{a}) + r_a^e(\tilde{c},\tilde{a})],$$

yields the expressions that are given on page 17:

$$\mathrm{tr}\,(\mathbf{J}) = r^* + \frac{\rho - r^*}{1-\eta\,(1-\theta)}, \qquad \det\,(\mathbf{J}) = -\frac{(\rho - r^*)\left\{\rho - [1+(\xi/\eta)]r^*\right\}}{[1-\eta\,(1-\theta)]\,(\xi/\eta)}.$$

5.15 The dependence of $|\mu_1|$ on ξ

On page 17 we state that, "It is straightforward to calculate that $\partial|\mu_1|/\partial\xi < 0$".

Proof: Since $\det(\mathbf{J}) < 0$, the solution for the negative root can be written as

$$\mu_1 = \frac{1}{2}\left[\operatorname{tr}(\mathbf{J}) - \sqrt{[\operatorname{tr}(\mathbf{J})]^2 + 4\left|\det(\mathbf{J})\right|}\right] < 0.$$

Writing $|\det(\mathbf{J})|$ as

$$|\det(\mathbf{J})| = \frac{\rho - r^*}{1 - \eta(1-\theta)}\left[\frac{\rho - r^*}{\xi/\eta} - r^*\right],$$

it is obvious that $\partial\left|\det(\mathbf{J})\right|/\partial\xi < 0$. Since tr $(\mathbf{J})$ is independent of ξ, it is clear that $\partial\mu_1/\partial\xi > 0$, which proves that $\partial\left|\mu_1\right|/\partial\xi < 0$.

5.16 The long-run effects of fiscal shocks (general framework)

The steady-state values $\tilde{a}$ and $\tilde{c}$ are implicitly determined by (31),

$$\tilde{c} = r^*\tilde{a} + \tilde{w} - (r^*b_0 + g), \qquad r^e(r^*, \tilde{c}, \tilde{a}) = \rho, \tag{80}$$

where

$$\tilde{w} = \tilde{w}(r^*, B), \qquad \tilde{w}_{r^*} = -\tilde{k} < 0, \qquad \tilde{w}_B = f(\tilde{k}) > 0, \tag{81}$$

holds according to (9). Differentiation of (80) with respect to g yields

$$\frac{\partial\tilde{c}}{\partial g} = r^*\frac{\partial\tilde{a}}{\partial g} - 1, \qquad r_c^e(\tilde{c},\tilde{a})\frac{\partial\tilde{c}}{\partial g} + r_a^e(\tilde{c},\tilde{a})\frac{\partial\tilde{a}}{\partial g} = 0.$$

Solving for $\partial\tilde{c}/\partial g$ and $\partial\tilde{a}/\partial g$, we obtain (32)

$$\frac{\partial\tilde{a}}{\partial g} = -\frac{r_c^e(\tilde{c},\tilde{a})}{\Psi} < 0, \qquad \frac{\partial\tilde{c}}{\partial g} = \frac{r_a^e(\tilde{c},\tilde{a})}{\Psi} = -\left[1 + \frac{r^* r_c^e(\tilde{c},\tilde{a})}{\Psi}\right] < -1,$$

where

$$\Psi = -\left[r^* r_c^e(\tilde{c},\tilde{a}) + r_a^e(\tilde{c},\tilde{a})\right] = -\frac{\det(\mathbf{J})}{\tilde{c}\sigma^e(\tilde{c})} > 0.$$

The inequalities given above follow from $r_c^e > 0$, $r_a^e < 0$ and the fact that we restrict attention to the case in which the steady state is a saddlepoint.

5.17 The long-run effects of productivity shocks (general framework)

Differentiation of (80) with respect to B yields

$$\frac{\partial\tilde{c}}{\partial B} = r^*\frac{\partial\tilde{a}}{\partial B} + \frac{\partial\tilde{w}}{\partial B}, \qquad r_c^e(\tilde{c},\tilde{a})\frac{\partial\tilde{c}}{\partial B} + r_a^e(\tilde{c},\tilde{a})\frac{\partial\tilde{a}}{\partial B} = 0.$$

Solving for $\partial\tilde{c}/\partial B$ and $\partial\tilde{a}/\partial B$, we find (34):

$$\frac{\partial\tilde{a}}{\partial B} = \frac{r_c^e(\tilde{c},\tilde{a})}{\Psi}\frac{\partial\tilde{w}}{\partial B} > 0, \quad \frac{\partial\tilde{c}}{\partial B} = \frac{-r_a^e(\tilde{c},\tilde{a})}{\Psi}\frac{\partial\tilde{w}}{\partial B} = \left[1 + \frac{r^* r_c^e(\tilde{c},\tilde{a})}{\Psi}\right]\frac{\partial\tilde{w}}{\partial B} > \frac{\partial\tilde{w}}{\partial B}.$$

5.18 The long-run effects of changes in r^* and ρ (general framework)

The long-run effects of changes in r^* and ρ are mentioned only in footnote 27. Differentiating (80) with respect to r^* and taking into account that $\tilde{w}_{r^*} = -\tilde{k}$ and $r_{r^*}^e = 1$, we obtain

$$\frac{\partial\tilde{c}}{\partial r^*} = \tilde{a} + r^*\frac{\partial\tilde{a}}{\partial r^*} - \tilde{k} - b_0, \qquad 1 + r_c^e(\tilde{c},\tilde{a})\frac{\partial\tilde{c}}{\partial r^*} + r_a^e(\tilde{c},\tilde{a})\frac{\partial\tilde{a}}{\partial r^*} = 0.$$

Solving for $\partial\tilde{c}/\partial r^*$ and $\partial\tilde{a}/\partial r^*$, we obtain

$$\frac{\partial\tilde{a}}{\partial r^*} = \frac{r_c^e(\tilde{c},\tilde{a})\left(\tilde{a} - b_0 - \tilde{k}\right) + 1}{\Psi}, \qquad \frac{\partial\tilde{c}}{\partial r^*} = -\frac{r_a^e(\tilde{c},\tilde{a})\left(\tilde{a} - b_0 - \tilde{k}\right) - r^*}{\Psi}.$$

Differentiation of (80) with respect to ρ yields

$$\frac{\partial\tilde{c}}{\partial\rho} = r^*\frac{\partial\tilde{a}}{\partial\rho}, \qquad r_c^e(\tilde{c},\tilde{a})\frac{\partial\tilde{c}}{\partial\rho} + r_a^e(\tilde{c},\tilde{a})\frac{\partial\tilde{a}}{\partial\rho} = 1.$$

Solving for $\partial\tilde{c}/\partial\rho$ and $\partial\tilde{a}/\partial\rho$, we find

$$\frac{\partial\tilde{a}}{\partial\rho} = -\frac{1}{\Psi} < 0, \qquad \frac{\partial\tilde{c}}{\partial\rho} = -\frac{r^*}{\Psi} < 0.$$

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